Fourier Cosine Transforms: Expressions with Power-Law Functions

No	*Original function*, $f(x)$	*Cosine transform*, $\check{f}_c(u) = \int_0^\infty f(x)\cos(ux)\,dx$
1	$\begin{cases} 1 & \text{if } 0 < x < a, \\ 0 & \text{if } a < x \end{cases}$	$\dfrac{1}{u}\sin(au)$
2	$\dfrac{1}{a+x}, \quad a > 0$	$-\sin(au)\,\mathrm{si}(au) - \cos(au)\,\mathrm{Ci}(au)$
3	$\dfrac{1}{a^2+x^2}, \quad a > 0$	$\dfrac{\pi}{2a}e^{-au}$
4	$\dfrac{1}{a^2-x^2}, \quad a > 0$	$\dfrac{\pi\sin(au)}{2u}$ (the integral is understood in the sense of Cauchy principal value)
5	$\dfrac{a}{a^2+(b+x)^2} + \dfrac{a}{a^2+(b-x)^2}$	$\pi e^{-au}\cos(bu)$
6	$\dfrac{b+x}{a^2+(b+x)^2} + \dfrac{b-x}{a^2+(b-x)^2}$	$\pi e^{-au}\sin(bu)$
7	$\dfrac{1}{a^4+x^4}, \quad a > 0$	$\frac{1}{2}\pi a^{-3}\exp\left(-\dfrac{au}{\sqrt{2}}\right)\sin\left(\dfrac{\pi}{4}+\dfrac{au}{\sqrt{2}}\right)$
8	$\dfrac{1}{(a^2+x^2)(b^2+x^2)}, \quad a, b > 0$	$\dfrac{\pi}{2}\,\dfrac{ae^{-bu}-be^{-au}}{ab(a^2-b^2)}$
9	$\dfrac{x^{2m}}{(x^2+a)^{n+1}}$, $n, m = 1, 2, \ldots;\ n+1 > m \geq 0$	$(-1)^{n+m}\dfrac{\pi}{2n!}\dfrac{\partial^n}{\partial a^n}\left(a^{1/\sqrt{m}}e^{-u\sqrt{a}}\right)$
10	$\dfrac{1}{\sqrt{x}}$	$\sqrt{\dfrac{\pi}{2u}}$
11	$\dfrac{1}{\sqrt{a^2+x^2}}$	$K_0(au)$
12	$(a^2+x^2)^{-1/2}\left[(a^2+x^2)^{1/2}+a\right]^{1/2}$	$(2u/\pi)^{-1/2}e^{-au}, \quad a > 0$
13	$x^{-\nu}, \quad 0 < \nu < 1$	$\sin\left(\frac{1}{2}\pi\nu\right)\Gamma(1-\nu)u^{\nu-1}$

Notation: $\mathrm{Ci}(z)$ is the integral cosine, $K_0(z)$ is the modified Bessel function of the second kind, $\Gamma(z)$ is the gamma function.

References

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