Fourier Cosine Transforms: Expressions with Exponential Functions

No	*Original function*, $f(x)$	*Cosine transform*, $\check{f}_c(u) = \int_0^\infty f(x)\cos(ux)\,dx$
1	e^{-ax}	$\dfrac{a}{a^2+u^2}$
2	$\dfrac{1}{x}\left(e^{-ax}-e^{-bx}\right)$	$\dfrac{1}{2}\ln\dfrac{b^2+u^2}{a^2+u^2}$
3	$\sqrt{x}e^{-ax}$	$\frac{1}{2}\sqrt{\pi}\,(a^2+u^2)^{-3/4}\cos\left(\frac{3}{2}\arctan\dfrac{u}{a}\right)$
4	$\dfrac{1}{\sqrt{x}}e^{-ax}$	$\sqrt{\dfrac{\pi}{2}}\left[\dfrac{a+(a^2+u^2)^{1/2}}{a^2+u^2}\right]^{1/2}$
5	$x^n e^{-ax}, \quad n=1,2,\ldots$	$\dfrac{a^{n+1}n!}{(a^2+u^2)^{n+1}}\sum\limits_{0\le 2k\le n+1}(-1)^k C_{n+1}^{2k}\left(\dfrac{u}{a}\right)^{2k}$
6	$x^{n-1/2}e^{-ax}, \quad n=1,2,\ldots$	$k_n u\dfrac{\partial^n}{\partial a^n}\dfrac{1}{r\sqrt{r-a}}$, where $r=\sqrt{a^2+u^2}$, $k_n=(-1)^n\sqrt{\pi/2}$
7	$x^{\nu-1}e^{-ax}$	$\Gamma(\nu)(a^2+u^2)^{-\nu/2}\cos\left(\nu\arctan\dfrac{u}{a}\right)$
8	$\dfrac{x}{e^{ax}-1}$	$\dfrac{1}{2u^2}-\dfrac{\pi^2}{2a^2\sinh^2(\pi a^{-1}u)}$
9	$\dfrac{1}{x}\left(\dfrac{1}{2}-\dfrac{1}{x}+\dfrac{1}{e^x-1}\right)$	$-\dfrac{1}{2}\ln\left(1-e^{-2\pi u}\right)$
10	$\exp(-ax^2)$	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{a}}\exp\left(-\dfrac{u^2}{4a}\right)$
11	$\dfrac{1}{\sqrt{x}}\exp\left(-\dfrac{a}{x}\right)$	$\sqrt{\dfrac{\pi}{2u}}\,e^{-\sqrt{2au}}\left[\cos\left(\sqrt{2au}\right)-\sin\left(\sqrt{2au}\right)\right]$
12	$\dfrac{1}{x\sqrt{x}}\exp\left(-\dfrac{a}{x}\right)$	$\sqrt{\dfrac{\pi}{a}}\,e^{-\sqrt{2au}}\cos\left(\sqrt{2au}\right)$

References

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