Fourier Cosine Transforms: Expressions with Logarithmic Functions

No	*Original function*, $f(x)$	*Cosine transform*, $\check{f}_c(u)=\int_0^\infty f(x)\cos(ux)\,dx$
1	$\begin{cases}\ln x & \text{if } 0<x<1,\\ 0 & \text{if } 1<x\end{cases}$	$-\dfrac{1}{u}\operatorname{Si}(u)$
2	$\dfrac{\ln x}{\sqrt{x}}$	$-\sqrt{\dfrac{\pi}{2u}}\left[\ln(4u)+\mathcal{C}+\dfrac{\pi}{2}\right]$, $\mathcal{C}=0.5772\ldots$ is the Euler constant
3	$x^{\nu-1}\ln x, \quad 0<\nu<1$	$\Gamma(\nu)\cos\left(\dfrac{\pi\nu}{2}\right)u^{-\nu}\left[\psi(\nu)-\dfrac{\pi}{2}\tan\left(\dfrac{\pi\nu}{2}\right)-\ln u\right]$
4	$\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert, \quad a>0$	$\dfrac{2}{u}\left[\cos(au)\operatorname{Si}(au)-\sin(au)\operatorname{Ci}(au)\right]$
5	$\ln\left(1+a^2/x^2\right), \quad a>0$	$\dfrac{\pi}{u}\left(1-e^{-au}\right)$
6	$\ln\dfrac{a^2+x^2}{b^2+x^2}, \quad a,b>0$	$\dfrac{\pi}{u}\left(e^{-bu}-e^{-au}\right)$
7	$e^{-ax}\ln x, \quad a>0$	$-\dfrac{aC+\frac{1}{2}a\ln(u^2+a^2)+u\arctan(u/a)}{u^2+a^2}$
8	$\ln\left(1+e^{-ax}\right), \quad a>0$	$\dfrac{a}{2u^2}-\dfrac{\pi}{2u\sinh\left(\pi a^{-1}u\right)}$
9	$\ln\left(1-e^{-ax}\right), \quad a>0$	$\dfrac{a}{2u^2}-\dfrac{\pi}{2u}\coth\left(\pi a^{-1}u\right)$

Notation: $\operatorname{Ci}(z)$ is the integral cosine, $\operatorname{Si}(z)$ is the integral sine, $\Gamma(z)$ is the gamma function, $\psi(z)$ is the logarithmic derivative of the gamma function.

References

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