Fourier Sine Transforms: Expressions with Power-Law Functions

No	*Original function*, $f(x)$	*Sine transform*, $\check{f}_s(u) = \int_0^\infty f(x)\sin(ux)\,dx$
1	$\begin{cases} 1 & \text{if } 0 < x < a, \\ 0 & \text{if } a < x \end{cases}$	$\frac{1}{u}\left[1-\cos(au)\right]$
2	$\frac{1}{x}$	$\frac{\pi}{2}$
3	$\frac{1}{a+x}, \quad a>0$	$\sin(au)\,\mathrm{Ci}(au) - \cos(au)\,\mathrm{si}(au)$
4	$\frac{x}{a^2+x^2}, \quad a>0$	$\frac{\pi}{2}e^{-au}$
5	$\frac{1}{x(a^2+x^2)}, \quad a>0$	$\frac{\pi}{2a^2}\left(1-e^{-au}\right)$
6	$\frac{a}{a^2+(x-b)^2} - \frac{a}{a^2+(x+b)^2}$	$\pi e^{-au}\sin(bu)$
7	$\frac{x+b}{a^2+(x+b)^2} - \frac{x-b}{a^2+(x-b)^2}$	$\pi e^{-au}\cos(bu)$
8	$\frac{x}{(x^2+a^2)^n}, \quad a>0,\ n=1,2,\ldots$	$\frac{\pi u e^{-au}}{2^{2n-2}(n-1)!\,a^{2n-3}}\sum_{k=0}^{n-2}\frac{(2n-k-4)!}{k!\,(n-k-2)!}(2au)^k$
9	$\frac{x^{2m+1}}{(x^2+a)^{n+1}}$, $n,m=0,1,\ldots;\ 0\le m\le n$	$(-1)^{n+m}\frac{\pi}{2n!}\frac{\partial^n}{\partial a^n}\left(a^m e^{-u\sqrt{a}}\right)$
10	$\frac{1}{\sqrt{x}}$	$\sqrt{\frac{\pi}{2u}}$
11	$\frac{1}{x\sqrt{x}}$	$\sqrt{2\pi u}$
12	$x(a^2+x^2)^{-3/2}$	$uK_0(au)$
13	$x^{-\nu}, \quad 0<\nu<2$	$\cos\left(\frac{1}{2}\pi\nu\right)\Gamma(1-\nu)u^{\nu-1}$

Notation: $\mathrm{Ci}(z)$ is the integral cosine, $\mathrm{si}(z) = \mathrm{Si}(z) - \frac{\pi}{2}$, $\mathrm{Si}(z)$ is the integral sine, $K_0(z)$ is the modified Bessel function of the second kind, $\Gamma(z)$ is the gamma function.

References

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