



Fourier Sine Transforms: Expressions with Exponential Functions

No	<i>Original function</i> , $f(x)$	<i>Sine transform</i> , $\check{f}_s(u) = \int_0^\infty f(x) \sin(ux) dx$
1	$e^{-ax}, a > 0$	$\frac{u}{a^2 + u^2}$
2	$x^n e^{-ax}, a > 0, n = 1, 2, \dots$	$n! \left(\frac{a}{a^2 + u^2}\right)^{n+1} \sum_{k=0}^{[n/2]} (-1)^k C_{n+1}^{2k+1} \left(\frac{u}{a}\right)^{2k+1}$
3	$\frac{1}{x} e^{-ax}, a > 0$	$\arctan \frac{u}{a}$
4	$\sqrt{x} e^{-ax}, a > 0$	$\frac{\sqrt{\pi}}{2} (a^2 + u^2)^{-3/4} \sin\left(\frac{3}{2} \arctan \frac{u}{a}\right)$
5	$\frac{1}{\sqrt{x}} e^{-ax}, a > 0$	$\sqrt{\frac{\pi}{2}} \frac{(\sqrt{a^2 + u^2} - a)^{1/2}}{\sqrt{a^2 + u^2}}$
6	$\frac{1}{x\sqrt{x}} e^{-ax}, a > 0$	$\sqrt{2\pi} (\sqrt{a^2 + u^2} - a)^{1/2}$
7	$x^{n-1/2} e^{-ax}, a > 0, n = 1, 2, \dots$	$(-1)^n \sqrt{\frac{\pi}{2}} \frac{\partial^n}{\partial a^n} \left[\frac{(\sqrt{a^2 + u^2} - a)^{1/2}}{\sqrt{a^2 + u^2}} \right]$
8	$x^{\nu-1} e^{-ax}, a > 0, \nu > -1$	$\Gamma(\nu) (a^2 + u^2)^{-\nu/2} \sin\left(\nu \arctan \frac{u}{a}\right)$
9	$\frac{1}{e^{ax} + 1}, a > 0$	$\frac{1}{2u} - \frac{\pi}{2a \sinh(\pi u/a)}$
10	$\frac{1}{e^{ax} - 1}, a > 0$	$\frac{\pi}{2a} \coth\left(\frac{\pi u}{a}\right) - \frac{1}{2u}$
11	$x \exp(-ax^2)$	$\frac{\sqrt{\pi}}{4a^{3/2}} u \exp\left(-\frac{u^2}{4a}\right)$
12	$\frac{1}{\sqrt{x}} \exp\left(-\frac{a}{x}\right)$	$\sqrt{\frac{\pi}{2u}} e^{-\sqrt{2au}} [\cos(\sqrt{2au}) + \sin(\sqrt{2au})]$
13	$\frac{1}{x\sqrt{x}} \exp\left(-\frac{a}{x}\right)$	$\sqrt{\frac{\pi}{a}} e^{-\sqrt{2au}} \sin(\sqrt{2au})$

References

Bateman, H. and Erdélyi, A., *Tables of Integral Transforms. Vols. 1 and 2,* McGraw-Hill Book Co., New York, 1954.
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Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations,* CRC Press, Boca Raton, 1998.