



Fourier Sine Transforms: Expressions with Hyperbolic Functions

No	<i>Original function</i> , $f(x)$	<i>Sine transform</i> , $\check{f}_s(u) = \int_0^\infty f(x) \sin(ux) dx$
1	$\frac{1}{\sinh(ax)}, \quad a > 0$	$\frac{\pi}{2a} \tanh\left(\frac{1}{2}\pi a^{-1}u\right)$
2	$\frac{x}{\sinh(ax)}, \quad a > 0$	$\frac{\pi^2 \sinh\left(\frac{1}{2}\pi a^{-1}u\right)}{4a^2 \cosh^2\left(\frac{1}{2}\pi a^{-1}u\right)}$
3	$\frac{1}{x} e^{-bx} \sinh(ax), \quad b > a $	$\frac{1}{2} \arctan\left(\frac{2au}{u^2 + b^2 - a^2}\right)$
4	$\frac{1}{x \cosh(ax)}, \quad a > 0$	$\arctan\left[\sinh\left(\frac{1}{2}\pi a^{-1}u\right)\right]$
5	$1 - \tanh\left(\frac{1}{2}ax\right), \quad a > 0$	$\frac{1}{u} - \frac{\pi}{a \sinh(\pi a^{-1}u)}$
6	$\coth\left(\frac{1}{2}ax\right) - 1, \quad a > 0$	$\frac{\pi}{a} \coth(\pi a^{-1}u) - \frac{1}{u}$
7	$\frac{\cosh(ax)}{\sinh(bx)}, \quad a < b$	$\frac{\pi}{2b} \frac{\sinh(\pi b^{-1}u)}{\cos(\pi a b^{-1}) + \cosh(\pi b^{-1}u)}$
8	$\frac{\sinh(ax)}{\cosh(bx)}, \quad a < b$	$\frac{\pi}{b} \frac{\sin\left(\frac{1}{2}\pi a b^{-1}\right) \sinh\left(\frac{1}{2}\pi b^{-1}u\right)}{\cos(\pi a b^{-1}) + \cosh(\pi b^{-1}u)}$

References

Bateman, H. and Erdélyi, A., *Tables of Integral Transforms. Vols. 1 and 2,* McGraw-Hill Book Co., New York, 1954.
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Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations* , CRC Press, Boca Raton, 1998.