Fourier Sine Transforms: Expressions with Trigonometric Functions

No	***Original function***, $f(x)$	***Sine transform***, $\check{f}_s(u) = \int_0^\infty f(x)\sin(ux)\,dx$
1	$\dfrac{\sin(ax)}{x}, \quad a>0$	$\dfrac{1}{2}\ln\left\lvert\dfrac{u+a}{u-a}\right\rvert$
2	$x^{\nu-1}\sin(ax), \quad a>0,\ -2<\nu<1$	$\pi\dfrac{\lvert u-a\rvert^{-\nu}-\lvert u+a\rvert^{-\nu}}{4\Gamma(1-\nu)\sin\left(\frac{1}{2}\pi\nu\right)}, \quad \nu\neq 0$
3	$\dfrac{\sin(ax)}{x^2+b^2}, \quad a,b>0$	$\begin{cases}\frac{1}{2}\pi b^{-1}e^{-ab}\sinh(bu) & \text{if } 0<u<a,\\ \frac{1}{2}\pi b^{-1}e^{-bu}\sinh(ab) & \text{if } u>a\end{cases}$
4	$\dfrac{\sin(\pi x)}{1-x^2}$	$\begin{cases}\sin u & \text{if } 0<u<\pi,\\ 0 & \text{if } u>\pi\end{cases}$
5	$e^{-ax}\sin(bx), \quad a>0$	$\dfrac{a}{2}\left[\dfrac{1}{a^2+(b-u)^2}-\dfrac{1}{a^2+(b+u)^2}\right]$
6	$x^{-1}e^{-ax}\sin(bx), \quad a>0$	$\dfrac{1}{4}\ln\dfrac{(u+b)^2+a^2}{(u-b)^2+a^2}$
7	$\exp\left(-ax^2\right)\sin(bx), \quad a>0$	$\dfrac{1}{2}\sqrt{\dfrac{\pi}{a}}\exp\left(-\dfrac{u^2+b^2}{4a}\right)\sinh\left(\dfrac{bu}{2a}\right)$
8	$\sin\left(\dfrac{a}{x}\right), \quad a>0$	$\dfrac{\pi\sqrt{a}}{2\sqrt{u}}J_1\left(2\sqrt{au}\right)$
9	$\dfrac{1}{\sqrt{x}}\sin\left(\dfrac{a}{x}\right), \quad a>0$	$\sqrt{\dfrac{\pi}{8u}}\left[\sin\left(2\sqrt{au}\right)-\cos\left(2\sqrt{au}\right)+\exp\left(-2\sqrt{au}\right)\right]$
10	$\exp\left(-a\sqrt{x}\right)\sin\left(a\sqrt{x}\right), \quad a>0$	$a\sqrt{\dfrac{\pi}{8}}\,u^{-3/2}\exp\left(-\dfrac{a^2}{2u}\right)$
11	$x^{\nu-1}\cos(ax), \quad a>0,\ \lvert\nu\rvert<1$	$\dfrac{\pi(u+a)^{-\nu}-\operatorname{sign}(u-a)\lvert u-a\rvert^{-\nu}}{4\Gamma(1-\nu)\cos\left(\frac{1}{2}\pi\nu\right)}$
12	$\dfrac{x\cos(ax)}{x^2+b^2}, \quad a,b>0$	$\begin{cases}-\frac{1}{2}\pi e^{-ab}\sinh(bu) & \text{if } u<a,\\ \frac{1}{2}\pi e^{-bu}\cosh(ab) & \text{if } u>a\end{cases}$
13	$\dfrac{1-\cos(ax)}{x^2}, \quad a>0$	$\dfrac{u}{2}\ln\left\lvert\dfrac{u^2-a^2}{u^2}\right\rvert+\dfrac{a}{2}\ln\left\lvert\dfrac{u+a}{u-a}\right\rvert$
14	$\dfrac{1}{\sqrt{x}}\cos\left(a\sqrt{x}\right)$	$\sqrt{\dfrac{\pi}{u}}\cos\left(\dfrac{a^2}{4u}+\dfrac{\pi}{4}\right)$

Notation: $J_1(z)$ is the Bessel function of the first kind, $\Gamma(z)$ is the gamma function.

References

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