Inverse Laplace Transforms: General Formulas

No	*Laplace transform*, $\widetilde{f}(p)$	*Inverse transform*, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$\widetilde{f}(p+a)$	$e^{-ax}f(x)$
2	$\widetilde{f}(ap),\quad a>0$	$\dfrac{1}{a}f\Bigl(\dfrac{x}{a}\Bigr)$
3	$\widetilde{f}(ap+b),\quad a>0$	$\dfrac{1}{a}\exp\Bigl(-\dfrac{b}{a}x\Bigr)f\Bigl(\dfrac{x}{a}\Bigr)$
4	$\widetilde{f}(p-a)+\widetilde{f}(p+a)$	$2f(x)\cosh(ax)$
5	$\widetilde{f}(p-a)-\widetilde{f}(p+a)$	$2f(x)\sinh(ax)$
6	$e^{-ap}\widetilde{f}(p),\quad a\ge 0$	$\begin{cases}0 & \text{if } 0\le x<a,\\ f(x-a) & \text{if } a<x.\end{cases}$
7	$p\widetilde{f}(p)$	$\dfrac{df(x)}{dx},\quad \text{if } f(+0)=0$
8	$\dfrac{1}{p+a}\widetilde{f}(p)$	$e^{-ax}\displaystyle\int_0^x e^{at}f(t)\,dt$
9	$\dfrac{1}{p^2}\widetilde{f}(p)$	$\displaystyle\int_0^x (x-t)f(t)\,dt$
10	$\dfrac{\widetilde{f}(p)}{p(p+a)}$	$\dfrac{1}{a}\displaystyle\int_0^x \bigl[1-e^{a(x-t)}\bigr]f(t)\,dt$
11	$\dfrac{\widetilde{f}(p)}{(p+a)^2}$	$\displaystyle\int_0^x (x-t)e^{-a(x-t)}f(t)\,dt$
12	$\dfrac{\widetilde{f}(p)}{(p+a)(p+b)}$	$\dfrac{1}{b-a}\displaystyle\int_0^x \bigl[e^{-a(x-t)}-e^{-b(x-t)}\bigr]f(t)\,dt$
13	$\dfrac{\widetilde{f}(p)}{(p+a)^2+b^2}$	$\dfrac{1}{b}\displaystyle\int_0^x e^{-a(x-t)}\sin\bigl[b(x-t)\bigr]f(t)\,dt$
14	$\dfrac{1}{p^n}\widetilde{f}(p),\quad n=1,\,2,\,\ldots$	$\dfrac{1}{(n-1)!}\displaystyle\int_0^x (x-t)^{n-1}f(t)\,dt$
15	$\widetilde{f}_1(p)\widetilde{f}_2(p)$	$\displaystyle\int_0^x f_1(t)f_2(x-t)\,dt$
16	$\dfrac{1}{\sqrt{p}}\widetilde{f}\Bigl(\dfrac{1}{p}\Bigr)$	$\displaystyle\int_0^\infty \frac{\cos\bigl(2\sqrt{xt}\,\bigr)}{\sqrt{\pi x}}f(t)\,dt$
17	$\dfrac{1}{p\sqrt{p}}\widetilde{f}\Bigl(\dfrac{1}{p}\Bigr)$	$\displaystyle\int_0^\infty \frac{\sin\bigl(2\sqrt{xt}\,\bigr)}{\sqrt{\pi t}}f(t)\,dt$

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18	$\dfrac{1}{p^{2\nu+1}}\widetilde{f}\Big(\dfrac{1}{p}\Big)$	$\displaystyle\int_0^\infty (x/t)^\nu J_{2\nu}\big(2\sqrt{xt}\,\big) f(t)\,dt$
19	$\dfrac{1}{p}\widetilde{f}\Big(\dfrac{1}{p}\Big)$	$\displaystyle\int_0^\infty J_0\big(2\sqrt{xt}\,\big) f(t)\,dt$
20	$\widetilde{f}\big(\sqrt{p^2+a^2}\,\big)$	$f(x) - a\displaystyle\int_0^x f\big(\sqrt{x^2-t^2}\,\big) J_1(at)\,dt$
21	$\widetilde{f}\big(\sqrt{p^2-a^2}\,\big)$	$f(x) + a\displaystyle\int_0^x f\big(\sqrt{x^2-t^2}\,\big) I_1(at)\,dt$
22	$\dfrac{\widetilde{f}\big(\sqrt{p^2+a^2}\,\big)}{\sqrt{p^2+a^2}}$	$\displaystyle\int_0^x J_0\big(a\sqrt{x^2-t^2}\,\big) f(t)\,dt$
23	$\dfrac{\widetilde{f}\big(\sqrt{p^2-a^2}\,\big)}{\sqrt{p^2-a^2}}$	$\displaystyle\int_0^x I_0\big(a\sqrt{x^2-t^2}\,\big) f(t)\,dt$
24	$\widetilde{f}(\ln p)$	$\displaystyle\int_0^\infty \frac{x^{t-1}}{\Gamma(t)} f(t)\,dt$
25	$\dfrac{d\widetilde{f}(p)}{dp}$	$-x f(x)$
26	$\dfrac{d^n\widetilde{f}(p)}{dp^n}$	$(-x)^n f(x)$
27	$p^n\dfrac{d^m\widetilde{f}(p)}{dp^m}, \quad m \ge n$	$(-1)^m \dfrac{d^n}{dx^n}\big[x^m f(x)\big]$
28	$\displaystyle\int_p^\infty \widetilde{f}(q)\,dq$	$\dfrac{1}{x} f(x)$

Notation: $J_\nu(z)$ is the Bessel function of the first kind, $I_\nu(z)$ is the modified Bessel function of the first kind, $\Gamma(z)$ is the gamma function,

References

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