Inverse Laplace Transforms: Expressions with Rational Functions

No	*Laplace transform*, $\widetilde{f}(p)$	*Inverse transform*, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$\dfrac{1}{p}$	1
2	$\dfrac{1}{p+a}$	e^{-ax}
3	$\dfrac{1}{p^2}$	x
4	$\dfrac{1}{p(p+a)}$	$\dfrac{1}{a}\left(1-e^{-ax}\right)$
5	$\dfrac{1}{(p+a)^2}$	xe^{-ax}
6	$\dfrac{p}{(p+a)^2}$	$(1-ax)e^{-ax}$
7	$\dfrac{1}{p^2-a^2}$	$\dfrac{1}{a}\sinh(ax)$
8	$\dfrac{p}{p^2-a^2}$	$\cosh(ax)$
9	$\dfrac{1}{(p+a)(p+b)}$	$\dfrac{1}{a-b}\left(e^{-bx}-e^{-ax}\right)$
10	$\dfrac{p}{(p+a)(p+b)}$	$\dfrac{1}{a-b}\left(ae^{-ax}-be^{-bx}\right)$
11	$\dfrac{1}{p^2+a^2}$	$\dfrac{1}{a}\sin(ax)$
12	$\dfrac{p}{p^2+a^2}$	$\cos(ax)$
13	$\dfrac{1}{(p+b)^2+a^2}$	$\dfrac{1}{a}e^{-bx}\sin(ax)$
14	$\dfrac{p}{(p+b)^2+a^2}$	$e^{-bx}\left[\cos(ax)-\dfrac{b}{a}\sin(ax)\right]$
15	$\dfrac{1}{p^n}, \quad n=1,\,2,\,\dots$	$\dfrac{1}{(n-1)!}x^{n-1}$
16	$\dfrac{1}{(p+a)^n}, \quad n=1,\,2,\,\dots$	$\dfrac{1}{(n-1)!}x^{n-1}e^{-ax}$

No	Laplace transform, $\widetilde{f}(p)$	Inverse transform, $f(x) = \dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
17	$\dfrac{1}{p(p+a)^n}, \quad n=1,\,2,\,\ldots$	$a^{-n}\bigl[1-e^{-ax}e_n(ax)\bigr]$, where $e_n(z) = 1 + \dfrac{z}{1!} + \cdots + \dfrac{z^n}{n!}$
18	$\dfrac{Q(p)}{P(p)}$, $P(p) = (p-a_1)\ldots(p-a_n)$; $Q(p)$ is a polynomial of degree $\le n-1$; $a_i \neq a_j$ if $i \neq j$	$\displaystyle\sum_{k=1}^{n}\frac{Q(a_k)}{P'(a_k)}\exp\bigl(a_k x\bigr)$, (the prime stand for the differentiation)

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