Inverse Laplace Transforms: Expressions with Square Roots

No	*Laplace transform*, $\widetilde{f}(p)$	*Inverse transform*, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$\dfrac{1}{\sqrt{p}}$	$\dfrac{1}{\sqrt{\pi x}}$
2	$\sqrt{p-a}-\sqrt{p-b}$	$\dfrac{e^{bx}-e^{ax}}{2\sqrt{\pi x^3}}$
3	$\dfrac{1}{\sqrt{p+a}}$	$\dfrac{1}{\sqrt{\pi x}}e^{-ax}$
4	$\sqrt{\dfrac{p+a}{p}}-1$	$\frac{1}{2}ae^{-ax/2}\left[I_1\left(\frac{1}{2}ax\right)+I_0\left(\frac{1}{2}ax\right)\right]$
5	$\dfrac{\sqrt{p+a}}{p+b}$	$\dfrac{e^{-ax}}{\sqrt{\pi x}}+(a-b)^{1/2}e^{-bx}\operatorname{erf}\left[(a-b)^{1/2}x^{1/2}\right]$
6	$\dfrac{1}{p\sqrt{p}}$	$2\sqrt{\dfrac{x}{\pi}}$
7	$\dfrac{1}{(p+a)\sqrt{p+b}}$	$(b-a)^{-1/2}e^{-ax}\operatorname{erf}\left[(b-a)^{1/2}x^{1/2}\right]$
8	$\dfrac{1}{\sqrt{p}\,(p-a)}$	$\dfrac{1}{\sqrt{a}}e^{ax}\operatorname{erf}\left(\sqrt{ax}\right)$
9	$\dfrac{1}{p^{3/2}(p-a)}$	$a^{-3/2}e^{ax}\operatorname{erf}\left(\sqrt{ax}\right)-2a^{-1}\pi^{-1/2}x^{1/2}$
10	$\dfrac{1}{\sqrt{p}+a}$	$\pi^{-1/2}x^{-1/2}-ae^{a^2x}\operatorname{erfc}\left(a\sqrt{x}\right)$
11	$\dfrac{a}{p\left(\sqrt{p}+a\right)}$	$1-e^{a^2x}\operatorname{erfc}\left(a\sqrt{x}\right)$
12	$\dfrac{1}{p+a\sqrt{p}}$	$e^{a^2x}\operatorname{erfc}\left(a\sqrt{x}\right)$
13	$p^{-n-1/2}, \quad n=1,\,2,\,\ldots$	$\dfrac{2^n}{1\cdot 3\ldots(2n-1)\sqrt{\pi}}\,x^{n-1/2}$
14	$(p+a)^{-n-1/2}$	$\dfrac{2^n}{1\cdot 3\ldots(2n-1)\sqrt{\pi}}\,x^{n-1/2}e^{-ax}$
15	$\dfrac{1}{\sqrt{p^2+a^2}}$	$J_0(ax)$
16	$\dfrac{1}{\sqrt{p^2-a^2}}$	$I_0(ax)$

No	***Laplace transform***, $\widetilde{f}(p)$	***Inverse transform***, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
17	$\dfrac{1}{\sqrt{p^2+ap+b}}$	$\exp\left(-\frac{1}{2}ax\right)J_0\left[\left(b-\frac{1}{4}a^2\right)^{1/2}x\right]$
18	$\left(\sqrt{p^2+a^2}+p\right)^{-n}$	$na^{-n}x^{-1}J_n(ax)$
19	$\left(\sqrt{p^2-a^2}+p\right)^{-n}$	$na^{-n}x^{-1}I_n(ax)$
20	$\left(p^2+a^2\right)^{-n-1/2}$	$\dfrac{(x/a)^n J_n(ax)}{1\cdot 3\cdot 5\ldots(2n-1)}$
21	$\left(p^2-a^2\right)^{-n-1/2}$	$\dfrac{(x/a)^n I_n(ax)}{1\cdot 3\cdot 5\ldots(2n-1)}$

Notation: $J_\nu(z)$ is the Bessel function of the first kind, $I_\nu(z)$ is the modified Bessel function of the first kind, erf z is the error function, erfc z is the complementary error function.

References

Bateman, H. and Erdélyi, A., *Tables of Integral Transforms. Vols. 1 and 2*, McGraw-Hill Book Co., New York, 1954.

Doetsch, G., *Einführung in Theorie und Anwendung der Laplace-Transformation*, Birkhäuser Verlag, Basel–Stuttgart, 1958.

Ditkin, V. A. and Prudnikov, A. P., *Integral Transforms and Operational Calculus*, Pergamon Press, New York, 1965.

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations* , CRC Press, Boca Raton, 1998.