Inverse Laplace Transforms: Expressions with Arbitrary Powers

No	***Laplace transform***, $\widetilde{f}(p)$	***Inverse transform***, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$(p+a)^{-\nu},\quad \nu>0$	$\dfrac{1}{\Gamma(\nu)}x^{\nu-1}e^{-ax}$
2	$\bigl[(p+a)^{1/2}+(p+b)^{1/2}\bigr]^{-2\nu},\quad \nu>0$	$\dfrac{\nu}{(a-b)^{\nu}}x^{-1}\exp\bigl[-\tfrac{1}{2}(a+b)x\bigr]I_{\nu}\bigl[\tfrac{1}{2}(a-b)x\bigr]$
3	$\bigl[(p+a)(p+b)\bigr]^{-\nu},\quad \nu>0$	$\dfrac{\sqrt{\pi}}{\Gamma(\nu)}\Bigl(\dfrac{x}{a-b}\Bigr)^{\nu-1/2}\exp\Bigl(-\dfrac{a+b}{2}x\Bigr)I_{\nu-1/2}\Bigl(\dfrac{a-b}{2}x\Bigr)$
4	$\bigl(p^2+a^2\bigr)^{-\nu-1/2},\quad \nu>-\frac{1}{2}$	$\dfrac{\sqrt{\pi}}{(2a)^{\nu}\Gamma(\nu+\frac{1}{2})}x^{\nu}J_{\nu}(ax)$
5	$\bigl(p^2-a^2\bigr)^{-\nu-1/2},\quad \nu>-\frac{1}{2}$	$\dfrac{\sqrt{\pi}}{(2a)^{\nu}\Gamma(\nu+\frac{1}{2})}x^{\nu}I_{\nu}(ax)$
6	$p\bigl(p^2+a^2\bigr)^{-\nu-1/2},\quad \nu>0$	$\dfrac{a\sqrt{\pi}}{(2a)^{\nu}\Gamma\bigl(\nu+\frac{1}{2}\bigr)}x^{\nu}J_{\nu-1}(ax)$
7	$p\bigl(p^2-a^2\bigr)^{-\nu-1/2},\quad \nu>0$	$\dfrac{a\sqrt{\pi}}{(2a)^{\nu}\Gamma\bigl(\nu+\frac{1}{2}\bigr)}x^{\nu}I_{\nu-1}(ax)$
8	$\bigl[(p^2+a^2)^{1/2}+p\bigr]^{-\nu},\quad \nu>0$	$\nu a^{-\nu}x^{-1}J_{\nu}(ax)$
9	$\bigl[(p^2-a^2)^{1/2}+p\bigr]^{-\nu},\quad \nu>0$	$\nu a^{-\nu}x^{-1}I_{\nu}(ax)$
10	$p\bigl[(p^2+a^2)^{1/2}+p\bigr]^{-\nu},\quad \nu>1$	$\nu a^{1-\nu}x^{-1}J_{\nu-1}(ax)-\nu(\nu+1)a^{-\nu}x^{-2}J_{\nu}(ax)$
11	$p\bigl[(p^2-a^2)^{1/2}+p\bigr]^{-\nu},\quad \nu>1$	$\nu a^{1-\nu}x^{-1}I_{\nu-1}(ax)-\nu(\nu+1)a^{-\nu}x^{-2}I_{\nu}(ax)$
12	$\dfrac{\bigl(\sqrt{p^2+a^2}+p\bigr)^{-\nu}}{\sqrt{p^2+a^2}},\quad \nu>-1$	$a^{-\nu}J_{\nu}(ax)$
13	$\dfrac{\bigl(\sqrt{p^2-a^2}+p\bigr)^{-\nu}}{\sqrt{p^2-a^2}},\quad \nu>-1$	$a^{-\nu}I_{\nu}(ax)$

Notation: $J_{\nu}(z)$ is the Bessel function of the first kind, $I_{\nu}(z)$ is the modified Bessel function of the first kind, $\Gamma(z)$ is the gamma function.

References

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