Inverse Laplace Transforms: Expressions with Exponential Functions

No	*Laplace transform*, $\widetilde{f}(p)$	*Inverse transform*, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$p^{-1}e^{-ap}, \quad a>0$	$\begin{cases} 0 & \text{if } 0<x<a, \\ 1 & \text{if } a<x. \end{cases}$
2	$p^{-1}\left(1-e^{-ap}\right), \quad a>0$	$\begin{cases} 1 & \text{if } 0<x<a, \\ 0 & \text{if } a<x. \end{cases}$
3	$p^{-1}\left(e^{-ap}-e^{-bp}\right), \quad 0\le a<b$	$\begin{cases} 0 & \text{if } 0<x<a, \\ 1 & \text{if } a<x<b, \\ 0 & \text{if } b<x. \end{cases}$
4	$p^{-2}\left(e^{-ap}-e^{-bp}\right), \quad 0\le a<b$	$\begin{cases} 0 & \text{if } 0<x<a, \\ x-a & \text{if } a<x<b, \\ b-a & \text{if } b<x. \end{cases}$
5	$(p+b)^{-1}e^{-ap}, \quad a>0$	$\begin{cases} 0 & \text{if } 0<x<a, \\ e^{-b(x-a)} & \text{if } a<x. \end{cases}$
6	$p^{-\nu}e^{-ap}, \quad \nu>0$	$\begin{cases} 0 & \text{if } 0<x<a, \\ \dfrac{(x-a)^{\nu-1}}{\Gamma(\nu)} & \text{if } a<x. \end{cases}$
7	$p^{-1}\left(e^{ap}-1\right)^{-1}, \quad a>0$	$f(x)=n \quad \text{if} \quad na<x<(n+1)a; \quad n=0,\,1,\,\ldots$
8	$e^{a/p}-1$	$\sqrt{\dfrac{a}{x}}\,I_1\left(2\sqrt{ax}\right)$
9	$p^{-1/2}e^{a/p}$	$\dfrac{1}{\sqrt{\pi x}}\cosh\left(2\sqrt{ax}\right)$
10	$p^{-3/2}e^{a/p}$	$\dfrac{1}{\sqrt{\pi a}}\sinh\left(2\sqrt{ax}\right)$
11	$p^{-\nu-1}e^{a/p}, \quad \nu>-1$	$(x/a)^{\nu/2}I_\nu\left(2\sqrt{ax}\right)$
12	$1-e^{-a/p}$	$\sqrt{\dfrac{a}{x}}\,J_1\left(2\sqrt{ax}\right)$
13	$p^{-1/2}e^{-a/p}$	$\dfrac{1}{\sqrt{\pi x}}\cos\left(2\sqrt{ax}\right)$
14	$p^{-3/2}e^{-a/p}$	$\dfrac{1}{\sqrt{\pi a}}\sin\left(2\sqrt{ax}\right)$
15	$p^{-\nu-1}e^{-a/p}, \quad \nu>-1$	$(x/a)^{\nu/2}J_\nu\left(2\sqrt{ax}\right)$
16	$\exp\left(-\sqrt{ap}\right), \quad a>0$	$\dfrac{\sqrt{a}}{2\sqrt{\pi}}\,x^{-3/2}\exp\left(-\dfrac{a}{4x}\right)$

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17	$\dfrac{1}{p}\exp\bigl(-\sqrt{ap}\,\bigr), \quad a\ge 0$	$\operatorname{erfc}\Bigl(\dfrac{\sqrt{a}}{2\sqrt{x}}\Bigr)$
18	$\dfrac{1}{\sqrt{p}}\exp\bigl(-\sqrt{ap}\,\bigr), \quad a\ge 0$	$\dfrac{1}{\sqrt{\pi x}}\exp\Bigl(-\dfrac{a}{4x}\Bigr)$
19	$\dfrac{1}{p\sqrt{p}}\exp\bigl(-\sqrt{ap}\,\bigr), \quad a\ge 0$	$\dfrac{2\sqrt{x}}{\sqrt{\pi}}\exp\Bigl(-\dfrac{a}{4x}\Bigr)-\sqrt{a}\operatorname{erfc}\Bigl(\dfrac{\sqrt{a}}{2\sqrt{x}}\Bigr)$
20	$\dfrac{\exp\bigl(-k\sqrt{p^2+a^2}\,\bigr)}{\sqrt{p^2+a^2}}, \quad k>0$	$\begin{cases} 0 & \text{if } 0<x<k, \\ J_0\bigl(a\sqrt{x^2-k^2}\,\bigr) & \text{if } k<x. \end{cases}$
21	$\dfrac{\exp\bigl(-k\sqrt{p^2-a^2}\,\bigr)}{\sqrt{p^2-a^2}}, \quad k>0$	$\begin{cases} 0 & \text{if } 0<x<k, \\ I_0\bigl(a\sqrt{x^2-k^2}\,\bigr) & \text{if } k<x. \end{cases}$

Notation: $\operatorname{erfc} z$ is the complementary error function, $J_\nu(z)$ is the Bessel function of the first kind, $I_\nu(z)$ is the modified Bessel function of the first kind, $\Gamma(z)$ is the gamma function.

References

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