Inverse Laplace Transforms: Expressions with Hyperbolic Functions

No	*Laplace transform*, $\tilde{f}(p)$	*Inverse transform*, $f(x) = \dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\tilde{f}(p)\,dp$
1	$\dfrac{1}{p\sinh(ap)}, \quad a>0$	$f(x)=2n$ if $a(2n-1)<x<a(2n+1)$; $n=0,\,1,\,2,\,\dots\quad (x>0)$
2	$\dfrac{1}{p^2\sinh(ap)}, \quad a>0$	$f(x)=2n(x-an)$ if $a(2n-1)<x<a(2n+1)$; $n=0,\,1,\,2,\,\dots\quad (x>0)$
3	$\dfrac{\sinh(a/p)}{\sqrt{p}}$	$\dfrac{1}{2\sqrt{\pi x}}\bigl[\cosh\bigl(2\sqrt{ax}\,\bigr)-\cos\bigl(2\sqrt{ax}\,\bigr)\bigr]$
4	$\dfrac{\sinh(a/p)}{p\sqrt{p}}$	$\dfrac{1}{2\sqrt{\pi a}}\bigl[\sinh\bigl(2\sqrt{ax}\,\bigr)-\sin\bigl(2\sqrt{ax}\,\bigr)\bigr]$
5	$p^{-\nu-1}\sinh(a/p), \quad \nu>-2$	$\frac{1}{2}(x/a)^{\nu/2}\bigl[I_\nu\bigl(2\sqrt{ax}\,\bigr)-J_\nu\bigl(2\sqrt{ax}\,\bigr)\bigr]$
6	$\dfrac{1}{p\cosh(ap)}, \quad a>0$	$f(x)=\begin{cases}0 & \text{if } a(4n-1)<x<a(4n+1),\\ 2 & \text{if } a(4n+1)<x<a(4n+3),\end{cases}$ $n=0,\,1,\,2,\,\dots\quad (x>0)$
7	$\dfrac{1}{p^2\cosh(ap)}, \quad a>0$	$x-(-1)^n(x-2an)$ if $2n-1<x/a<2n+1$; $n=0,\,1,\,2,\,\dots\quad (x>0)$
8	$\dfrac{\cosh(a/p)}{\sqrt{p}}$	$\dfrac{1}{2\sqrt{\pi x}}\bigl[\cosh\bigl(2\sqrt{ax}\,\bigr)+\cos\bigl(2\sqrt{ax}\,\bigr)\bigr]$
9	$\dfrac{\cosh(a/p)}{p\sqrt{p}}$	$\dfrac{1}{2\sqrt{\pi a}}\bigl[\sinh\bigl(2\sqrt{ax}\,\bigr)+\sin\bigl(2\sqrt{ax}\,\bigr)\bigr]$
10	$p^{-\nu-1}\cosh(a/p), \quad \nu>-1$	$\frac{1}{2}(x/a)^{\nu/2}\bigl[I_\nu\bigl(2\sqrt{ax}\,\bigr)+J_\nu\bigl(2\sqrt{ax}\,\bigr)\bigr]$
11	$\dfrac{1}{p}\tanh(ap), \quad a>0$	$f(x)=(-1)^{n-1}$ if $2a(n-1)<x<2an$; $n=1,\,2,\,\dots$
12	$\dfrac{1}{p}\coth(ap), \quad a>0$	$f(x)=(2n-1)$ if $2a(n-1)<x<2an$; $n=1,\,2,\,\dots$

Notation: $J_\nu(z)$ is the Bessel function of the first kind, $I_\nu(z)$ is the modified Bessel function of the first kind.

References

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