Inverse Laplace Transforms: Expressions with Logarithmic Functions

No	*Laplace transform*, $\widetilde{f}(p)$	*Inverse transform*, $f(x)=\dfrac{1}{2\pi i}\displaystyle\int_{c-i\infty}^{c+i\infty} e^{px}\widetilde{f}(p)\,dp$
1	$\dfrac{1}{p}\ln p$	$-\ln x-\mathcal{C}$, $\mathcal{C}=0.5772\ldots$ is the Euler constant
2	$p^{-n-1}\ln p$	$\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln x-\mathcal{C}\right)\dfrac{x^n}{n!}$, $\mathcal{C}=0.5772\ldots$ is the Euler constant
3	$p^{-n-1/2}\ln p$	$k_n\left[2+\frac{2}{3}+\frac{2}{5}+\cdots+\frac{2}{2n-1}-\ln(4x)-\mathcal{C}\right]x^{n-1/2}$, $k_n=\dfrac{2^n}{1\cdot3\cdot5\ldots(2n-1)\sqrt{\pi}}$, $\mathcal{C}=0.5772\ldots$
4	$p^{-\nu}\ln p, \quad \nu>0$	$\dfrac{1}{\Gamma(\nu)}x^{\nu-1}\left[\psi(\nu)-\ln x\right]$, $\psi(\nu)$ is the logarithmic derivative of the gamma function
5	$\dfrac{1}{p}(\ln p)^2$	$(\ln x+\mathcal{C})^2-\frac{1}{6}\pi^2$, $\mathcal{C}=0.5772\ldots$
6	$\dfrac{1}{p^2}(\ln p)^2$	$x\left[(\ln x+\mathcal{C}-1)^2+1-\frac{1}{6}\pi^2\right]$
7	$\ln\dfrac{p+b}{p+a}$	$\dfrac{1}{x}\left(e^{-ax}-e^{-bx}\right)$
8	$\ln\dfrac{p^2+b^2}{p^2+a^2}$	$\dfrac{2}{x}\left[\cos(ax)-\cos(bx)\right]$
9	$p\ln\dfrac{p^2+b^2}{p^2+a^2}$	$\dfrac{2}{x}\left[\cos(bx)+bx\sin(bx)-\cos(ax)-ax\sin(ax)\right]$
10	$\ln\dfrac{(p+a)^2+k^2}{(p+b)^2+k^2}$	$\dfrac{2}{x}\cos(kx)(e^{-bx}-e^{-ax})$
11	$p\ln\left(\dfrac{1}{p}\sqrt{p^2+a^2}\right)$	$\dfrac{1}{x^2}\left[\cos(ax)-1\right]+\dfrac{a}{x}\sin(ax)$
12	$p\ln\left(\dfrac{1}{p}\sqrt{p^2-a^2}\right)$	$\dfrac{1}{x^2}\left[\cosh(ax)-1\right]-\dfrac{a}{x}\sinh(ax)$

References

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