Laplace Transforms: General Formulas

No	*Original function*, $f(x)$	*Laplace transform*, $\widetilde{f}(p)=\int_0^\infty e^{-px}f(x)\,dx$
1	$af_1(x)+bf_2(x)$	$a\widetilde{f}_1(p)+b\widetilde{f}_2(p)$
2	$f(x/a),\quad a>0$	$a\widetilde{f}(ap)$
3	$\begin{cases}0 & \text{if } 0<x<a,\\ f(x-a) & \text{if } a<x,\end{cases}$	$e^{-ap}\widetilde{f}(p)$
4	$x^n f(x);\quad n=1,\,2,\,\ldots$	$(-1)^n\dfrac{d^n}{dp^n}\widetilde{f}(p)$
5	$e^{ax}f(x)$	$\widetilde{f}(p-a)$
6	$\sinh(ax)f(x)$	$\frac{1}{2}\left[\widetilde{f}(p-a)-\widetilde{f}(p+a)\right]$
7	$\cosh(ax)f(x)$	$\frac{1}{2}\left[\widetilde{f}(p-a)+\widetilde{f}(p+a)\right]$
8	$\sin(\omega x)f(x)$	$-\frac{i}{2}\left[\widetilde{f}(p-i\omega)-\widetilde{f}(p+i\omega)\right],\quad i^2=-1$
9	$\cos(\omega x)f(x)$	$\frac{1}{2}\left[\widetilde{f}(p-i\omega)+\widetilde{f}(p+i\omega)\right],\quad i^2=-1$
10	$f(x+a)=f(x)$ (periodic function)	$\dfrac{1}{1-e^{ap}}\displaystyle\int_0^a f(x)e^{-px}\,dx$
11	$f'_x(x)$	$p\widetilde{f}(p)-f(+0)$
12	$f_x^{(n)}(x)$	$p^n\widetilde{f}(p)-\displaystyle\sum_{k=1}^{n}p^{n-k}f_x^{(k-1)}(+0)$
13	$x^m f_x^{(n)}(x),\quad m\ge n$	$\left(-\dfrac{d}{dp}\right)^m\left[p^n\widetilde{f}(p)\right]$
14	$\dfrac{d^n}{dx^n}\left[x^m f(x)\right],\quad m\ge n$	$(-1)^m p^n\dfrac{d^m}{dp^m}\widetilde{f}(p)$
15	$\displaystyle\int_0^x f(t)\,dt$	$\dfrac{\widetilde{f}(p)}{p}$
16	$\displaystyle\int_0^x (x-t)f(t)\,dt$	$\dfrac{1}{p^2}\widetilde{f}(p)$
17	$\displaystyle\int_0^x (x-t)^\nu f(t)\,dt,\qquad \nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}\widetilde{f}(p),\quad \Gamma(\nu)$ is the gamma function
18	$\displaystyle\int_0^x e^{-a(x-t)}f(t)\,dt$	$\dfrac{1}{p+a}\widetilde{f}(p)$
19	$\displaystyle\int_0^x \sinh\left[a(x-t)\right]f(t)\,dt$	$\dfrac{a\widetilde{f}(p)}{p^2-a^2}$

No	***Original function***, $f(x)$	***Laplace transform***, $\widetilde{f}(p)=\int_0^\infty e^{-px}f(x)\,dx$
20	$\int_0^x \sin\big[a(x-t)\big]f(t)\,dt$	$\dfrac{a\widetilde{f}(p)}{p^2+a^2}$
21	$\int_0^x f_1(t)f_2(x-t)\,dt$	$\widetilde{f}_1(p)\widetilde{f}_2(p)$
22	$\int_0^\infty \dfrac{1}{\sqrt{\pi x}}\exp\Big(-\dfrac{t^2}{4x}\Big)f(t)\,dt$	$\dfrac{1}{\sqrt{p}}\widetilde{f}\big(\sqrt{p}\big)$
23	$\int_0^\infty \dfrac{t}{2\sqrt{\pi x^3}}\exp\Big(-\dfrac{t^2}{4x}\Big)f(t)\,dt$	$\widetilde{f}\big(\sqrt{p}\big)$
24	$f(x)-a\int_0^x f\big(\sqrt{x^2-t^2}\,\big)J_1(at)\,dt$	$\widetilde{f}\big(\sqrt{p^2+a^2}\,\big)$
25	$f(x)+a\int_0^x f\big(\sqrt{x^2-t^2}\,\big)I_1(at)\,dt$	$\widetilde{f}\big(\sqrt{p^2-a^2}\,\big)$

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