Laplace Transforms: Expressions with Power-Law Functions

No	*Original function*, $f(x)$	*Laplace transform*, $\widetilde{f}(p)=\int_0^\infty e^{-px}f(x)\,dx$
1	1	$\dfrac{1}{p}$
2	$\begin{cases}0 & \text{if } 0<x<a,\\ 1 & \text{if } a<x<b,\\ 0 & \text{if } b<x.\end{cases}$	$\dfrac{1}{p}\left(e^{-ap}-e^{-bp}\right)$
3	x	$\dfrac{1}{p^2}$
4	$\dfrac{1}{x+a}$	$-e^{ap}\operatorname{Ei}(-ap)$
5	$x^n,\quad n=1,\,2,\,\ldots$	$\dfrac{n!}{p^{n+1}}$
6	$x^{n-1/2},\quad n=1,\,2,\,\ldots$	$\dfrac{1\cdot 3\ldots(2n-1)\sqrt{\pi}}{2^n p^{n+1/2}}$
7	$\dfrac{1}{\sqrt{x+a}}$	$\sqrt{\dfrac{\pi}{p}}\,e^{ap}\operatorname{erfc}\left(\sqrt{ap}\right)$
8	$\dfrac{\sqrt{x}}{x+a}$	$\sqrt{\dfrac{\pi}{p}}-\pi\sqrt{a}\,e^{ap}\operatorname{erfc}\left(\sqrt{ap}\right)$
9	$(x+a)^{-3/2}$	$2a^{-1/2}-2(\pi p)^{1/2}e^{ap}\operatorname{erfc}\left(\sqrt{ap}\right)$
10	$x^{1/2}(x+a)^{-1}$	$(\pi/p)^{1/2}-\pi a^{1/2}e^{ap}\operatorname{erfc}\left(\sqrt{ap}\right)$
11	$x^{-1/2}(x+a)^{-1}$	$\pi a^{-1/2}e^{ap}\operatorname{erfc}\left(\sqrt{ap}\right)$
12	$x^\nu,\quad \nu>-1$	$\Gamma(\nu+1)p^{-\nu-1}$
13	$(x+a)^\nu,\quad \nu>-1$	$p^{-\nu-1}e^{-ap}\Gamma(\nu+1,ap)$
14	$x^\nu(x+a)^{-1},\quad \nu>-1$	$ke^{ap}\Gamma(-\nu,ap),\quad k=a^\nu\Gamma(\nu+1)$

Notation: $\operatorname{Ei}(z)$ is the integral exponent, $\operatorname{erfc} z$ is the complementary error function, $\Gamma(\nu)$ is the gamma function, $\Gamma(\nu,z)$ is incomplete the gamma function.

References

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