Laplace Transforms: Expressions with Exponential Functions

No	*Original function*, $f(x)$	*Laplace transform*, $\widetilde{f}(p)=\int_0^\infty e^{-px}f(x)\,dx$
1	e^{-ax}	$(p+a)^{-1}$
2	xe^{-ax}	$(p+a)^{-2}$
3	$x^{\nu-1}e^{-ax}, \quad \nu>0$	$\Gamma(\nu)(p+a)^{-\nu}$
4	$\frac{1}{x}\left(e^{-ax}-e^{-bx}\right)$	$\ln(p+b)-\ln(p+a)$
5	$\frac{1}{x^2}\left(1-e^{-ax}\right)^2$	$(p+2a)\ln(p+2a)+p\ln p-2(p+a)\ln(p+a)$
6	$\exp\left(-ax^2\right), \quad a>0$	$(\pi b)^{1/2}\exp\left(bp^2\right)\operatorname{erfc}(p\sqrt{b}), \quad a=\frac{1}{4b}$
7	$x\exp\left(-ax^2\right)$	$2b-2\pi^{1/2}b^{3/2}p\operatorname{erfc}(p\sqrt{b}), \quad a=\frac{1}{4b}$
8	$\exp(-a/x), \quad a\geq 0$	$2\sqrt{a/p}K_1\left(2\sqrt{ap}\right)$
9	$\sqrt{x}\exp(-a/x), \quad a\geq 0$	$\frac{1}{2}\sqrt{\pi/p^3}\left(1+2\sqrt{ap}\right)\exp\left(-2\sqrt{ap}\right)$
10	$\frac{1}{\sqrt{x}}\exp(-a/x), \quad a\geq 0$	$\sqrt{\pi/p}\exp\left(-2\sqrt{ap}\right)$
11	$\frac{1}{x\sqrt{x}}\exp(-a/x), \quad a>0$	$\sqrt{\pi/a}\exp\left(-2\sqrt{ap}\right)$
12	$x^{\nu-1}\exp(-a/x), \quad a>0$	$2(a/p)^{\nu/2}K_\nu\left(2\sqrt{ap}\right)$
13	$\exp\left(-2\sqrt{ax}\right)$	$p^{-1}-(\pi a)^{1/2}p^{-3/2}e^{a/p}\operatorname{erfc}\left(\sqrt{a/p}\right)$
14	$\frac{1}{\sqrt{x}}\exp\left(-2\sqrt{ax}\right)$	$(\pi/p)^{1/2}e^{a/p}\operatorname{erfc}\left(\sqrt{a/p}\right)$

Notation: $\Gamma(\nu)$ is the gamma function, $\operatorname{erfc} z$ is the complementary error function, $K_\nu(z)$ is the modified Bessel function of the second kind.

References

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