Laplace Transforms: Expressions with Hyperbolic Functions

No	*Original function*, $f(x)$	*Laplace transform*, $\widetilde{f}(p)=\int_0^\infty e^{-px}f(x)\,dx$
1	$\sinh(ax)$	$\dfrac{a}{p^2-a^2}$
2	$\sinh^2(ax)$	$\dfrac{2a^2}{p^3-4a^2p}$
3	$\dfrac{1}{x}\sinh(ax)$	$\dfrac{1}{2}\ln\dfrac{p+a}{p-a}$
4	$x^{\nu-1}\sinh(ax), \quad \nu>-1$	$\frac{1}{2}\Gamma(\nu)\left[(p-a)^{-\nu}-(p+a)^{-\nu}\right]$
5	$\sinh\left(2\sqrt{ax}\right)$	$\dfrac{\sqrt{\pi a}}{p\sqrt{p}}e^{a/p}$
6	$\sqrt{x}\sinh\left(2\sqrt{ax}\right)$	$\pi^{1/2}p^{-5/2}\left(\frac{1}{2}p+a\right)e^{a/p}\operatorname{erf}\left(\sqrt{a/p}\right)-a^{1/2}p^{-2}$
7	$\dfrac{1}{\sqrt{x}}\sinh\left(2\sqrt{ax}\right)$	$\pi^{1/2}p^{-1/2}e^{a/p}\operatorname{erf}\left(\sqrt{a/p}\right)$
8	$\dfrac{1}{\sqrt{x}}\sinh^2\left(\sqrt{ax}\right)$	$\frac{1}{2}\pi^{1/2}p^{-1/2}\left(e^{a/p}-1\right)$
9	$\cosh(ax)$	$\dfrac{p}{p^2-a^2}$
10	$\cosh^2(ax)$	$\dfrac{p^2-2a^2}{p^3-4a^2p}$
11	$x^{\nu-1}\cosh(ax), \quad \nu>0$	$\frac{1}{2}\Gamma(\nu)\left[(p-a)^{-\nu}+(p+a)^{-\nu}\right]$
12	$\cosh\left(2\sqrt{ax}\right)$	$\dfrac{1}{p}+\dfrac{\sqrt{\pi a}}{p\sqrt{p}}e^{a/p}\operatorname{erf}\left(\sqrt{a/p}\right)$
13	$\sqrt{x}\cosh\left(2\sqrt{ax}\right)$	$\pi^{1/2}p^{-5/2}\left(\frac{1}{2}p+a\right)e^{a/p}$
14	$\dfrac{1}{\sqrt{x}}\cosh\left(2\sqrt{ax}\right)$	$\pi^{1/2}p^{-1/2}e^{a/p}$
15	$\dfrac{1}{\sqrt{x}}\cosh^2\left(\sqrt{ax}\right)$	$\frac{1}{2}\pi^{1/2}p^{-1/2}\left(e^{a/p}+1\right)$

Notation: $\Gamma(\nu)$ is the gamma function, erfc z is the complementary error function.

References

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