



Laplace Transforms: Expressions with Error Functions

No	<i>Original function</i> , $f(x)$	<i>Laplace transform</i> , $\tilde{f}(p) = \int_0^\infty e^{-px} f(x) dx$
1	$\operatorname{erf}(ax)$	$\frac{1}{p} \exp(b^2 p^2) \operatorname{erfc}(bp), \quad b = \frac{1}{2a}$
2	$\operatorname{erf}(\sqrt{ax})$	$\frac{\sqrt{a}}{p\sqrt{p+a}}$
3	$e^{ax} \operatorname{erf}(\sqrt{ax})$	$\frac{\sqrt{a}}{\sqrt{p}(p-a)}$
4	$\operatorname{erf}\left(\frac{1}{2}\sqrt{a/x}\right)$	$\frac{1}{p} [1 - \exp(-\sqrt{ap})]$
5	$\operatorname{erfc}(\sqrt{ax})$	$\frac{\sqrt{p+a} - \sqrt{a}}{p\sqrt{p+a}}$
6	$e^{ax} \operatorname{erfc}(\sqrt{ax})$	$\frac{1}{p + \sqrt{ap}}$
7	$\operatorname{erfc}\left(\frac{1}{2}\sqrt{a/x}\right)$	$\frac{1}{p} \exp(-\sqrt{ap})$

Notation: $\operatorname{erf} z$ is the error function and $\operatorname{erfc} z$ is the complementary error function.

References

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