Mellin Transforms: General Formulas

No	*Original function*, $f(x)$	*Mellin transform*, $\hat{f}(s)=\int_0^\infty f(x)x^{s-1}\,dx$
1	$af_1(x)+bf_2(x)$	$a\hat{f}_1(s)+b\hat{f}_2(s)$
2	$f(ax),\ a>0$	$a^{-s}\hat{f}(s)$
3	$x^a f(x)$	$\hat{f}(s+a)$
4	$f(1/x)$	$\hat{f}(-s)$
5	$f\left(x^\beta\right),\ \ \beta>0$	$\frac{1}{\beta}\hat{f}\left(\frac{s}{\beta}\right)$
6	$f\left(x^{-\beta}\right),\ \ \beta>0$	$\frac{1}{\beta}\hat{f}\left(-\frac{s}{\beta}\right)$
7	$x^\lambda f\left(ax^\beta\right),\ \ a,\beta>0$	$\frac{1}{\beta}a^{-\frac{s+\lambda}{\beta}}\hat{f}\left(\frac{s+\lambda}{\beta}\right)$
8	$x^\lambda f\left(ax^{-\beta}\right),\ \ a,\beta>0$	$\frac{1}{\beta}a^{\frac{s+\lambda}{\beta}}\hat{f}\left(-\frac{s+\lambda}{\beta}\right)$
9	$f'_x(x)$	$-(s-1)\hat{f}(s-1)$
10	$xf'_x(x)$	$-s\hat{f}(s)$
11	$f_x^{(n)}(x)$	$(-1)^n\frac{\Gamma(s)}{\Gamma(s-n)}\hat{f}(s-n)$
12	$\left(x\frac{d}{dx}\right)^n f(x)$	$(-1)^n s^n\hat{f}(s)$
13	$\left(\frac{d}{dx}x\right)^n f(x)$	$(-1)^n(s-1)^n\hat{f}(s)$
14	$x^\alpha\int_0^\infty t^\beta f_1(xt)f_2(t)\,dt$	$\hat{f}_1(s+\alpha)\hat{f}_2(1-s-\alpha+\beta)$
15	$x^\alpha\int_0^\infty t^\beta f_1\left(\frac{x}{t}\right)f_2(t)\,dt$	$\hat{f}_1(s+\alpha)\hat{f}_2(s+\alpha+\beta+1)$

Notation: $\Gamma(z)$ is the gamma function.

References

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