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Asymptotic theory of two-phase gas–solid flow through a vertical tube at moderate pressure gradient

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Abstract

Based on the equations, constitutive relations and boundary conditions of the kinetic theory of colliding particles in a gas–solid suspension, the approximate theory of the steady, developed vertical flow of a gas-particulate mixture is developed for the case of moderate gas pressure gradient in a vertical tube. The basic equations and boundary conditions show a singular behaviour of the solution of the problem at the wall. The method of matched asymptotic expansions is applied to develop a boundary layer-type theory for the flow parameters of the particulate phase. The basic equations in the bulk flow are reduced to a system of two ordinary integrodifferential equations for the particle-phase concentration and mean kinetic energy of particle velocity fluctuations (particle-phase pseudotemperature). The distributions of the particle concentration and velocity are found in both the bulk and the boundary layer. The solution shows the bifurcation of flow parameters, and an explicit criterion is derived to identify a range of the given macroscopic parameters corresponding to upward or downward particulate flow. The integrated parameters (total fluxes of the gas and particle phase) are calculated.

Keywords: Kinetic theory of gas-particulate suspensions; Interparticle collisions; Multiphase flow; Two-phase gas-particulate flow; Granular flow

1. Introduction

The two-fluid model based on the kinetic theory of the colliding particles in a suspension (see, for example, [1,2]) has been successfully applied for a numerical study

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of two-phase flow in vertical tubes in [3–5]. The numerical results, giving a two-phase flow pattern (particularly the particle volume fraction and velocity distributions), clearly show the phenomenon of the formation of a dense layer of the particle phase at the wall, while the gas–solid mixture in the bulk of two-phase flow remains dilute. This phenomenon is of key importance for the analysis of the structure of the gas-particulate suspension in vertical tubes. It has also been shown in [6] that a 'microscopic' mechanism for formation of such a layer can be linked with the formation of clusters of solid particles and their disintegration at the wall.

Based on the numerical results showing that the behaviour of the solution near the wall is singular, we can expect that further progress to be made by an application of analytical perturbation methods. The dimensionless formulation of the equations and boundary conditions of the collisional version of the two-fluid model involves a parameter related to a balance between the flux and the rate of dissipation of the kinetic energy of the microscale random motion of particles at the wall. This parameter is proportional to the particle diameter scaled by the tube radius and, therefore, is very small in most cases of interest. It enters the mathematical formulation of the problem in a way typical for the theory of singular perturbations, so that an application of the matched asymptotic expansion technique is expected to be the most relevant for the analytical treatment of the problem.

An attempt to apply such a technique for an analysis of the fully developed two-phase vertical flow has been made in [7]. However, the consideration given in the cited work was limited by the case of very high gas pressure gradients. Below the approximate asymptotic theory is developed for the case of intermediate pressure gradients, the latter being of major theoretical and practical interest.

2. Basic equations

We consider the developed plane-parallel steady flow of a suspension of interacting solid particles in a gas in a vertical cylindrical tube. Solid particles are assumed to be spherical and of the same size, and the gas density is assumed to be much lower than the density of solid material. The version of the kinetic theory of a suspension of solid particles in a gas [2], in which the mechanism of interparticle interaction is assumed in the form of direct particle–particle collisions, is applied to formulate the basic equations, constitutive relations and boundary conditions of the two-phase flow. With the above assumptions these equations are the equations of mass and momentum conservation of the gas and particulate phase and the equation of conservation of the energy of microscale random motion of solid particles (i.e. the particle-phase pseudotemperature).

In case of steady plane-parallel developed flow of a suspension in a cylindrical tube, the radial components of the gas and particle velocity vanish, and these equations reduce to (see [3–5])

$$\varepsilon\Pi + \rho_g \varepsilon g - b(\alpha, |v - u|)(v - u) = 0\,, \tag{1}$$

$$\frac{1}{r}\frac{d}{dr}(r\sigma_{rz}) + \alpha\Pi + \rho\alpha g + b(\alpha, |v-u|)(v-u) = 0\,, \tag{2}$$

$$\frac{d}{dr}\sigma_{rr} = 0\,, \tag{3}$$

$$\sigma_{rz}\frac{dv}{dr} + \frac{1}{r}\frac{d(rq)}{dr} + \gamma = 0\,, \tag{4}$$

where Eqs. (1) and (2) represent the axial components of momentum conservation equations of the gas and particulate phase, respectively (the former is valid since $\rho_g \ll \rho$, where ρ_g and ρ are the densities of the gas and solid material respectively), Eq. (3) the radial component of the momentum conservation equation of the particle phase, and Eq. (4) the equation of conservation of the kinetic energy of the microscale random particulate motion. In Eqs. (1)–(4) r and z are the radial and vertical coordinate, respectively, α is the particle concentration (particle-phase volume fraction), $\varepsilon = 1-\alpha$ the voidage, v and u are the vertical components of the particulate phase and gas velocity, respectively, g is the gravity, $\Pi = dp/dz$ the pressure gradient, p the gas pressure, $\boldsymbol{\sigma}$ the effective stress tensor of the particulate phase, $b(\alpha, |v-u|)$ the viscous drag coefficient, and γ and q are, respectively, the rate of dissipation and the flux of the energy of microscale random motion of solid particles. The closure of system of equations (1)–(4) is given below by the constitutive relations for $\boldsymbol{\sigma}$, γ, and q following from the kinetic theory of interparticle collisions. For the developed plane-parallel flow the pressure gradient must be assumed a given constant parameter: $\Pi = dp/dz = \text{const}$.

Below, instead of the system (1)–(4) we consider the system of equations (3), (4) and the following equation of momentum conservation of the mixture obtained by the summation of Eqs. (1) and (2):

$$\frac{1}{r}\frac{d}{dr}(r\sigma_{rz}) + \Pi + (\alpha\rho + \varepsilon\rho_g)g = 0\,. \tag{5}$$

To consider the closure for the system of equations (3)–(5), we start with the constitutive relations for the components of the effective stress tensor $\boldsymbol{\sigma}$ of the particulate phase. The effective stresses in the particulate phase are induced by the random particulate motion, the latter created by interparticle collisions; see [2]. Based on the kinetic theory of a suspension of colliding particles in a gas [1,2], the components of the stress tensor can be expressed through the pseudotemperature E of random particulate motion. For the considered plane-parallel developed flow in a vertical tube, these components are (see also [3,5])

$$\sigma_{rz} = -D_p\rho\Phi(\alpha)\sqrt{E}\,dv/dr\,, \tag{6}$$

$$\sigma_{rr} = p^* = \rho S(\alpha)E\,, \tag{7}$$

where D_p is the particle diameter, p^* can be understood as the effective pressure in the particle phase (see also [8]). The functions $\Phi(\alpha)$ and $S(\alpha)$ are found in [2,4] (see also [5]) in the form

$$\Phi(\alpha) = \frac{5\sqrt{\pi}}{96}\left[\frac{1}{g_0(\alpha)}\left(1+\frac{8}{5}\alpha g_0(\alpha)\right)^2 + \frac{3\cdot 256}{25\pi}\alpha^2 g_0(\alpha)\right], \tag{8a}$$

$$S(\alpha) = \alpha\,[1+4\alpha g_0(\alpha)], \tag{8b}$$

where $g_0(\alpha)$ is the Carnahan–Starling radial distribution function

$$g_0(\alpha) = \frac{1}{1-(\alpha/\bar{\alpha})^{1/3}}, \tag{9}$$

with $\bar{\alpha} \simeq 0.65$ the maximum volume fraction of the random packing of spherical particles in a suspension.

In the framework of the collision theory the constitutive relation for the flux of the 'pseudothermal' energy q can be written in the form [2]

$$q = -D_p\rho\Psi(\alpha)\sqrt{E}\,dE/dr, \tag{10}$$

where $\Psi(\alpha)$ is given by the relation

$$\Psi(\alpha) = \frac{25\sqrt{\pi}}{128}\frac{1}{g_0(\alpha)}\left[\left(1+\frac{12}{5}\alpha g_0(\alpha)\right)^2 + \frac{512}{25\sqrt{\pi}}\alpha^2 g_0^2(\alpha)\right]. \tag{11}$$

To close the system of equations (3)–(5) we also need the constitutive relation for the rate of dissipation γ. Such a relation is given in [2] in the form $\gamma = \Omega(\alpha)E^{3/2}$. However, to simplify the following analysis we neglect the dissipation in the particulate phase ($\gamma = 0$), which is often justified in case of relatively dilute suspension. (For the rapid two-phase flow in vertical tubes α hardly exceeds 0.1–0.2 at the wall and 0.02–0.05 at the centre. Two-phase flows that are dilute in comparison with conventional dense fluidized beds, where dissipation in the particulate phase plays an important role, are considered below. We note, however, that the rate of dissipation in the particulate phase can be incorporated into the analysis given below).

Now we formulate the system of boundary conditions. At the axis of the tube we obviously have the conditions of symmetry:

$$dv/dr = dE/dr = 0 \quad \text{at } r = 0 \tag{12}$$

(the condition of symmetry for α then follows automatically from Eq. (3) and the constitutive relation (7)).

The general form of the boundary condition for the particulate phase velocity at the wall has been obtained in [9]. However, according to [3–5] the above-mentioned condition can be considerably simplified in case of rapid gas-particulate flow in a vertical tube. In this case the particle velocity at the centre considerably exceeds the velocity of solid particles at the wall, so that the boundary condition can be formulated in the approximate form

$$v = 0 \quad \text{at } r = R, \tag{13}$$

where R is the tube radius.

The boundary condition for the particle-phase pseudotemperature obtained in [9] (see also [2,4,5]) is of the form

$$w\alpha g_0(\alpha)E + D_p \Psi(\alpha)\, dE/dr = 0 \quad \text{at } r = R\,, \tag{14}$$

where

$$w = \frac{\sqrt{3}\pi}{4\bar{\alpha}}(1 - e_w^2)\,, \tag{15}$$

e_w is the coefficient of restitution for particle–wall collisions. It is natural to assume particle–wall collisions to be inelastic, so that $w \neq 0$ (in fact the assumption that particle–wall collisions are perfectly diffuse is practically always close to reality, so that we can safely set $e_w = 0$ and $w = \sqrt{3}\pi/(4\bar{\alpha}) \simeq 2.09$). Condition (15) actually represents the balance between the flux of the pseudothermal energy of the microscale random motion of the particulate phase and the rate of dissipation created by inelastic particle–wall collisions.

The system of equations, constitutive relations and boundary conditions is not yet closed; for closure we must, naturally, assume a given total mass flux of solid particles. Such a condition would, however, cause a certain inconvenience in solving the problem and interpreting the results. It is convenient to replace the integral condition for the total flux by the condition

$$\alpha = \alpha_+ \quad \text{at } r = R\,, \tag{16}$$

where the particle concentration at the wall α_+ is assumed to be given. As soon as the solution of the problem (3)–(5), (12)–(14) and (16) has been found, the total mass flux J can be calculated as a function of α_+ and, therefore, α_+ will be determined as a function of J to solve a problem in its original form.

With the above constitutive relations and boundary conditions, Eqs. (3)–(5) represent a close system for the unknowns v, α and E.

To introduce the dimensionless variables we note that for the considered problem the most convenient scales of length, velocity and solid pseudotemperature are, respectively,

$$R, \quad \sqrt{\frac{gR^2}{D_p}}, \quad \frac{gR^2}{D_p}\,. \tag{17}$$

Neglecting the density ratio $\rho_g/\rho \ll 1$ we obtain the following system of dimensionless equations:

$$\frac{1}{r}\frac{d}{dr}\left(r\Phi(\alpha)\sqrt{E}\,\frac{dv}{dr}\right) = -G + \alpha\,, \tag{18}$$

$$\frac{d[S(\alpha)E]}{dr} = 0\,, \tag{19}$$

$$\frac{1}{\Phi(\alpha)\sqrt{E}}\frac{1}{r}\frac{d}{dr}\left(r\Psi(\alpha)\sqrt{E}\,\frac{dE}{dr}\right) + \left(\frac{dv}{dr}\right)^2 = 0\,, \tag{20}$$

where the same symbols are now used for the dimensionless variables, $0 \leqslant r < 1$. The dimensionless pressure gradient G, given by

$$G = -\Pi/(\rho g)\,, \tag{21}$$

is the only parameter entering the basic equations (18)–(20).

Obviously, the conditions of symmetry (12) remain unchanged, while instead of (13) and (16) we now have

$$v = 0 \quad \text{and} \quad \alpha = \alpha_+ \quad \text{at } r = 1\,. \tag{22}$$

The dimensionless form of the condition (14) is as follows:

$$\beta \frac{dE}{dr} + E = 0 \quad \text{at } r = 1\,, \tag{23}$$

where we have introduced the dimensionless parameter

$$\beta = \frac{D_\mathrm{p}}{R}\, \frac{\Psi(\alpha_+)}{w\alpha_+ g_0(\alpha_+)}\,. \tag{24}$$

Since $D_\mathrm{p}/R \ll 1$ and $w \neq 0$, the parameter β will be small provided the particle concentration at the wall is not zero. (Indeed, the suspension at the wall is always dense – see the analysis below.) Since the coefficient of the first-order derivative in (23) is a small parameter, we have a singular perturbation problem which can be treated by the method of matching asymptotic expansions (see, for example, [10]). The solution of the problem (18)–(20), (12), (22)–(23) is then expected to show a boundary-layer-type behaviour; two domains can be identified, i.e. a thin boundary layer near the wall where the particle concentration is expected to be relatively high, and the dilute bulk (the existence of such domains is definitely confirmed by results of numerical studies of the collision model of gas-particulate suspension in a vertical tube, see [3–5]).

To proceed with an asymptotic analysis of the problem we must estimate first the thickness of the boundary layer. From Eq. (19) as $r \to 1$ we find

$$\left(\frac{d\alpha}{dr}\right)_{r=1} = -\frac{1}{E_+}\left(\frac{dE}{dr}\right)_{r=1}\left[\frac{d\ln S(\alpha_+)}{d\alpha_+}\right]^{-1}, \tag{25}$$

where E_+ is the particle-phase pseudotemperature at the wall. From (25) and the boundary condition (23) we find the gradient of the particle concentration at the wall in the form

$$\left(\frac{d\alpha}{dr}\right)_{r=1} = \left[\beta\,\frac{d\ln S(\alpha_+)}{d\alpha_+}\right]^{-1}. \tag{26}$$

It should be noted that the RHS of (26) depends only on α_+ and the parameter β. The thickness of the particulate boundary layer can then be estimated from (26) as

$$\delta = \frac{\alpha_+}{(d\alpha/dr)_{r=1}} = \beta\alpha_+\frac{d\ln S(\alpha_+)}{d\alpha_+} = \frac{D_\mathrm{p}}{R}\,\frac{\alpha_+\Psi(\alpha_+)}{wg_0(\alpha_+)}\,\frac{d\ln S(\alpha_+)}{d\alpha_+} = O(\beta) \tag{27}$$

provided $w \neq 0$ and α_+ is not too close to the concentration of the maximum random packing $\bar{\alpha}$ (see (9) and (11)).

The particle velocity gradient can be explicitly expressed in terms of α and E from Eq. (18) with the aid of the first boundary condition in (12) as follows:

$$\Phi(\alpha)\sqrt{E}\,\frac{dv}{dr} = -\frac{Gr}{2} + \frac{1}{r}\int_0^r \alpha r\,dr \tag{28}$$

so that the closed system of Eqs. (18)–(20) reduces to the system of two equations for the unknown functions E and α:

$$\frac{\Phi(\alpha)\sqrt{E}}{r}\frac{d}{dr}\left(r\Psi(\alpha)\sqrt{E}\,\frac{dE}{dr}\right) + \left\{-\frac{Gr}{2} + \frac{1}{r}\int_0^r \alpha r\,dr\right\}^2 = 0\,, \tag{29}$$

$$\frac{d[S(\alpha)E]}{dr} = 0 \tag{30}$$

subject to the second boundary condition in (12), the second condition in (22), and the condition (23).

We also need to specify an order of magnitude of the dimensionless pressure gradient G. An obvious choice is $G = O(1)$; such a magnitude of the pressure gradient has been used for the numerical analysis in [3–5] and asymptotical analysis in [7]. This case, however, corresponds to very high *dimensional* pressure gradients and, consequently, particle velocities. When $G = O(1)$, α can be neglected compared with G in the RHS of the equation of momentum conservation of the particulate phase (18). Obviously, for very low pressure gradients, only a downward motion of solid particles is possible ($G \ll \alpha$ in Eq. (18)); this type of flow, however, is not of a considerable interest. The case of major theoretical and practical interest is characterized by intermediate pressure gradients when both upward and downward flow of the particulate phase is possible (and both terms in RHS of Eq. (18) are of the same order of magnitude). We, therefore, must specify the order of magnitude of the dimensionless pressure gradient G in terms of small parameter β:

$$G = O(\beta^k) = o(1) \qquad (k > 0)\,. \tag{31}$$

This will be done in the next section.

3. Asymptotic analysis

The solution of the problem will be found by means of the method of matched asymptotic expansions (see, for example, [10]) with respect to the small parameter β. It will be sufficient to analyse only principal terms of asymptotic expansions of E, α and v. The following two domains are considered within the two-phase flow field: the inner region (i.e. the particulate boundary layer), $1 - r \leqslant O(\beta)$, in which the flow parameters are denoted as α^*, E^* and v^*, and the outer region (the bulk), $0 \leqslant r < 1 - O(\beta)$, in which we use the notations α_*, E_* and v_* for the flow parameters. In each of these domains the principal terms of the asymptotic expansions of the solution are sought.

Results of numerical studies [3,5] lead us to expect that the radial distribution of the particle concentration is such that, being of the order of unity at the wall, it decreases to the very small value in the bulk. It is, therefore, reasonable to assume

$$\alpha_+ = O(1) . \tag{32}$$

The boundary condition in the form (23) also suggests that E is small within the boundary layer, so that at the wall

$$E|_{r=1} = E_+ = O(\beta^\nu) , \tag{33}$$

where $\nu > 0$, while in the bulk Eq. (29) shows that the particle concentration is low, $\alpha_* = o(1)$, and $E_* = O(1)$. The value of ν will be found in the process of solving the problem (29)–(30), (12), (22)–(23).

Integrating Eq. (30) we obtain

$$S(\alpha)E = C = \text{const} \quad \text{for} \quad 0 \leqslant r \leqslant 1 . \tag{34}$$

The constant C is to be found in the process of solving the above-mentioned problem. From (33) and (32) it follows immediately that $C = O(\beta^\nu) = o(1)$.

It is important to note that for small α the function $S(\alpha)$ assumes an asymptotic behaviour (see (8b))

$$S(\alpha) = \alpha \quad \text{as } \alpha \to 0 , \tag{35}$$

and the relation (34) takes a simple form $\alpha E = C$.

We start with a formulation of the problem within the boundary layer, $1 - O(\beta) \leqslant r \leqslant 1$. Introducing the stretched coordinate

$$x = \beta^{-1}(1 - r) \tag{36}$$

such that $0 \leqslant x \leqslant \infty$ ($0 \leqslant x \leqslant O(1)$ within the boundary layer) and neglecting the terms of the orders higher than $O(1)$, from (19) and (20) we obtain the boundary layer equations in the form

$$\frac{d}{dx}\left[\Psi(\alpha^*)\sqrt{E^*}\,\frac{dE^*}{dx}\right] = 0, \qquad \frac{d[S(\alpha^*)E^*]}{dx} = 0 . \tag{37}$$

The boundary conditions for the system of equations (37) must be formulated at the wall:

$$\alpha^* = \alpha_+ \quad \text{and} \quad E^* = dE^*/dx \quad \text{at} \quad x = 0 . \tag{38}$$

The boundary conditions (38) are not sufficient to find a solution of the equations (37), and the condition of matching of the solution of (37) with the solution in the bulk must be added. However, it is shown below that the distribution of the particle concentration in the boundary layer can be found irrespective of the solution in the outer region.

The equations (37) have the first integrals which can be written in the form

$$\Psi(\alpha^*)\,d(E^*)^{3/2}/dx = C_1\,, \qquad S^{3/2}(\alpha^*)(E^*)^{3/2} = C_2\,, \tag{39}$$

where C_1 = const and C_2 = const. Using the boundary conditions (38) we find the following relation between the constants:

$$C_2 = \frac{2}{3}\frac{S^{3/2}(\alpha_+)}{\Psi(\alpha_+)}\,C_1\,. \tag{40}$$

Expressing $(E^*)^{3/2}$ through α^* by means of the second relation in (39) and substituting the result into the first equation in (39) we obtain the following equation for the distribution of particle concentration within the boundary layer:

$$\frac{d\alpha^*}{dx} + \mu(\alpha_+)\lambda(\alpha^*) = 0\,, \tag{41}$$

where

$$\mu(\alpha) = \frac{\Psi(\alpha)}{S^{3/2}(\alpha)}, \qquad \lambda(\alpha) = \frac{S^{5/2}(\alpha)}{\Psi(\alpha)}\left(\frac{dS}{d\alpha}\right)^{-1}. \tag{42}$$

Eq. (41) is to be solved subject to the first boundary condition in (38). It is seen immediately that the particle concentration distribution in the boundary layer depends only on the single parameter α_+ (particle concentration at the wall), and $\alpha^* \to 0$ as $x \to \infty$. This enables us to find an asymptotic behaviour of the particle concentration at the outer boundary of the inner region (boundary layer) and, hence, to derive a matching condition.

To achieve this, we note that $\Psi(\alpha) \simeq \Psi(0)$ and $S(\alpha) \simeq \alpha$ for small α, so that, using (11), we find the asymptotic behaviour of $\lambda(\alpha)$ in the form

$$\lambda(\alpha) = \frac{32}{75\sqrt{\pi}}\,\alpha^2\sqrt{\alpha} \quad \text{as } \alpha \to 0\,. \tag{43}$$

It then follows from Eq. (41) that

$$\alpha^* = \left[\frac{3\Psi(0)S^{3/2}(\alpha_+)}{2\Psi(\alpha_+)}\right]^{2/3} x^{-2/3} \quad \text{as } x \to \infty\,. \tag{44}$$

Relation (35) then shows the order of magnitude of the particle concentration in the outer region (the bulk):

$$\alpha_* = O(\beta^{2/3})\,. \tag{45}$$

This conclusion is consistent with the remark made above concerning the low magnitude of particle concentration at the centre of the tube.

It must also be noted that the simple, approximate, form of the particle concentration distribution in the boundary layer can be obtained in the case when the particle volume fraction at the wall is sufficiently low. In this case both $\Psi(\alpha_*)$ and $\Psi(\alpha_+)$ can be approximated by their value at $\alpha = 0$, i.e. $\Psi(\alpha) \simeq \Psi(0)$, and $S(\alpha_*)$ and $S(\alpha_+)$ by α_* and α_+, respectively (see (11) and (8b)), so that, from (41) and (42), we obtain

$$\alpha^* = \alpha_+(1 + \tfrac{2}{3}x)^{-2/3} . \tag{46}$$

This relation can be used for $\alpha_+ \leqslant 0.15$, what is a realistic value for many practical applications.

We do not give here the, rather trivial, graphic illustration of the concentration distribution in the boundary layer; the solution is represented by smooth curves emerging from α_+ and decreasing with x, and is very well described by the asymptotic behaviour (44) and approximate formula (46).

We now return to the analysis of the distributions of the parameters α and E in the bulk. First of all we formulate a basic assumption regarding the order of magnitude of the dimensionless pressure gradient G. Requiring now that both the terms on the RHS of Eq. (18) be of the same order, we assume

$$G = O(\beta^{2/3}) . \tag{47}$$

It was already mentioned that larger values of G correspond to very high pressure gradients, while from Eq. (18) it follows that a higher order of magnitude of G with respect to the parameter β corresponds only to a downward motion of the particle phase; the latter case is of a minor theoretical and practical interest. Assumption (47) is used below to analyse the most relevant case when the gas pressure gradient is not extremely high, but also not very low, so that both upward and downward flow of solid particles is possible. For the purpose of the following analysis we introduce the parameter (scaled dimensionless pressure gradient) Γ by the relation

$$G = \beta^{2/3}\Gamma \tag{48}$$

such that

$$\Gamma = O(1) . \tag{49}$$

Now we estimate the order of magnitude of the particle-phase pseudotemperature in the bulk (outer region), E_*. Assuming all terms of Eq. (29) to be of the same order of magnitude, from (45) and (47) we find

$$E_* = O(\beta^{2/3}) . \tag{50}$$

Using (35), the constant C in (34) is found to be of the order of $\beta^{4/3}$. Since α_* is small, the leading asymptotic terms of the functions $\Phi(\alpha)$ and $\Psi(\alpha)$ are

$$\Phi(0) = \frac{5\sqrt{\pi}}{96} \simeq 0.0923 , \qquad \Psi(0) = \frac{25\sqrt{\pi}}{128} \simeq 0.346 , \tag{51}$$

respectively. Introducing now the scaled particle concentration φ and pseudotemperature ζ by the relations

$$\alpha_* = \tfrac{1}{2}\beta^{2/3}\Gamma\,\varphi\,, \qquad E_* = \frac{\beta^{2/3}\Gamma}{2\sqrt{\Phi(0)\Psi(0)}}\,\zeta \tag{52}$$

so that $\varphi = O(1)$ and $\zeta = O(1)$, and using again the approximation (35), we reduce Eqs. (29) and (30) in the bulk to

$$\frac{\sqrt{\zeta}}{r}\frac{d}{dr}\left(r\sqrt{\zeta}\,\frac{d\zeta}{dr}\right) + \left(-r + \frac{1}{r}\int_0^r \varphi r\,dr\right)^2 = 0\,, \tag{53}$$

$$\frac{d(\varphi\zeta)}{dr} = 0 \tag{54}$$

within the interval $0 \leqslant r < 1 - O(\beta)$. The first integral of (54) obviously is

$$\varphi\zeta = h = \text{const}\,, \tag{55}$$

where the constant h is related to the constant C in (34) as follows:

$$h = C\,\frac{4\Phi^{1/2}(0)\Psi^{1/2}(0)}{\beta^{4/3}\Gamma^2} = O(1)\,. \tag{56}$$

Now we can formulate the problem in the outer region $0 \leqslant r < 1 - O(\beta)$ (the bulk) as the following equation for the scaled pseudotemperature:

$$\frac{\sqrt{\zeta}}{r}\frac{d}{dr}\left(r\sqrt{\zeta}\,\frac{d\zeta}{dr}\right) + \left(-r + \frac{h}{r}\int_0^r \frac{r\,dr}{\zeta}\right)^2 = 0 \tag{57}$$

subject to the boundary condition

$$d\zeta/dr = 0 \quad \text{at } r = 0\,. \tag{58}$$

We emphasize that the constant h is not yet known, and that the last boundary condition (58) is not sufficient to solve Eq. (57). To find the missing boundary condition and to determine h in terms of the given macroscopic parameters of the flow, we need to formulate the condition of asymptotical matching of the solution of the problem (57)–(58) in the outer region with the leading term of the pseudotemperature distribution in the boundary layer found from the problem (37)–(38). First of all we find the asymptotic behaviour of the particle-phase pseudotemperature at the outer boundary of the inner region (boundary layer). Since $\alpha^* \to 0$ as $x \to \infty$, expressing from (34) the pseudotemperature in the boundary layer in the form $E^* = C/S(\alpha^*)$, representing $S(\alpha^*)$ in the asymptotic form (35), and using the relation (56) between the constants C and h yields

$$E^* = h\,\frac{3^{2/3}\beta^{4/3}\Gamma^2\Psi^{2/3}(\alpha_+)}{\Phi^{1/2}(0)\Psi^{5/6}(0)S^{2/3}(\alpha_+)}\,x^{2/3} \quad \text{as } x \to \infty\,. \tag{59}$$

In particular, this means that the particle-phase pseudotemperature is of the order $\beta^{4/3}$ within the boundary layer, and is of the order $\beta^{2/3}$ in the bulk.

To find the unknown constant h we require, as $r \to 1$, the particle pseudotemperature in the outer region, E_*, given by (52), to match the asymptotic behaviour of the particle pseudotemperature in the boundary layer as $x \to \infty$. This, however, means that $\zeta \to 0$ as $r \to 1$, so instead of the equation for the pseudotemperature formulated within the interval $0 \leqslant r \leqslant 1 - O(\beta)$ subject to the conditions of symmetry and asymptotical matching we can consider Eq. (57) within the whole interval $0 \leqslant r \leqslant 1$ subject to the boundary conditions

$$\zeta = 0 \quad \text{at } r = 1, \qquad d\zeta/dr = 0 \quad \text{at } r = 0\,. \tag{60}$$

The asymptotic analysis of the problem (57), (60) in the vicinity of $r = 1$ shows that

$$\zeta = A(h)(1-r)^{2/3} \quad \text{as } r \to 1 \tag{61}$$

for any value of h in the interval $0 \leqslant h < \infty$. Here the function $A(h)$ must be calculated from the numerical solution of the problem (57), (60).

The numerical solution of this problem gives two branches of the curve $A(h)$ shown in Fig. 1; the 'upper' branch exists for $h \leqslant h_c \simeq 0.15$ ($A(h_c) \simeq 0.35$), while the 'lower' branch is represented by the continuous monotonic curve in the interval $0 \leqslant h < \infty$. These two branches represent two different solutions of the problem (57), (60), where $h = h_c$ is the bifurcation point, and, therefore, correspond to two different flow regimes potentially existing for $h \leqslant h_c \simeq 0.15$. The physical interpretation of the both regimes will be given later in this paper.

Since the asymptotic behaviour of E^* as $x \to \infty$, given by (59), must coincide with the asymptotic behaviour of E_* as $r \to 1$, given by (61) together with the second relation in (52), we find the following relation:

$$\frac{h}{A(h)} = \tau\,, \tag{62}$$

where the parameter τ is a known function of the macroscopic parameters β, G, and α_+, namely,

$$\tau(\beta, G, \alpha_+) = \frac{\beta^{2/3}}{G} \frac{2^{5/3}\Psi^{1/6}(0)}{3^{2/3}} \frac{S(\alpha_+)}{\Psi^{2/3}(\alpha_+)} \simeq 1.279\, \frac{\beta^{2/3}}{G} \frac{S(\alpha_+)}{\Psi^{2/3}(\alpha_+)}\,. \tag{63}$$

The parameter τ plays a key role in the following analysis. Since the function $A(h)$ is already determined by the results given in Fig. 1, Eq. (62) gives h and A as functions of the known parameter τ, e.g. $h(\tau)$ and $A(\tau)$, shown in Figs. 2 and 3, respectively. Both $h(\tau)$ and $A(\tau)$ have two branches each for $\tau \leqslant \tau_c \simeq 0.43$; the values of h and A at the bifurcation point are $h_c \simeq 0.15$ and $A_c \simeq 0.35$. It is important to note that two branches exist simultaneously at relatively low values of τ.

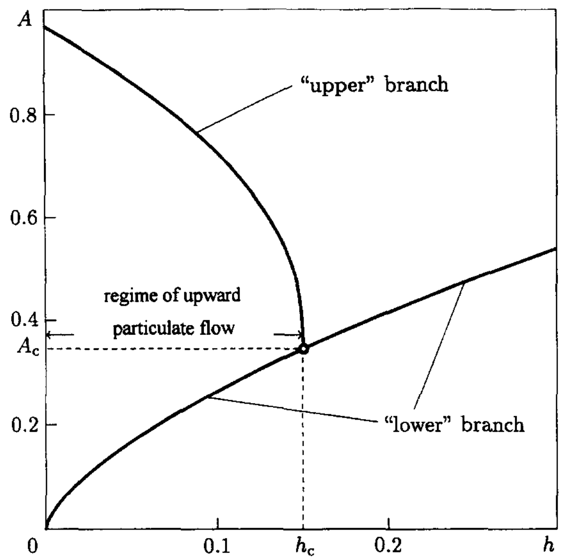

Fig. 1. Function $A(h)$ ('upper' branch corresponds to the regime of upward particulate flow); $h_c \simeq 0.15$, $A_c \simeq 0.35$.

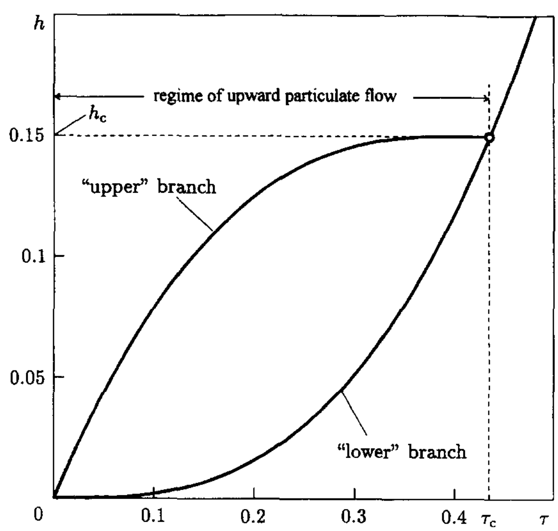

Fig. 2. Function $h(\tau)$ ('upper' branch corresponds to the regime of upward particulate flow); $\tau_c \simeq 0.43$.

From the definitions of the parameters β and G (see (24) and (21)) it then follows that two branches of the solution exist either in the case of relatively high pressure gradients or in the case of a very large tube radius (compared with the particle size).

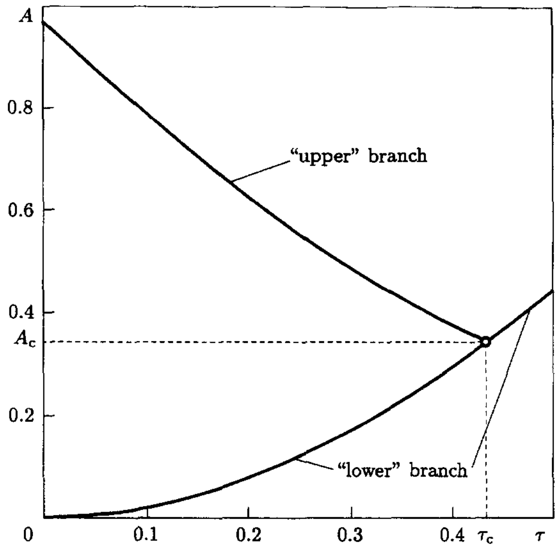

Fig. 3. Function $A(\tau)$ ('upper' branch corresponds to the regime of upward particulate flow).

The (scaled) pseudotemperature distribution, being the solution of the problem (57), (60), and the (scaled) particle concentration distribution in the bulk can now be represented as $\zeta(r,\tau)$ and

$$\varphi(r,\tau) = \frac{h(\tau)}{\zeta(r,\tau)}, \tag{64}$$

respectively. These functions are illustrated in Figs. 4 and 5, respectively, for various τ, where figures a correspond to the 'lower' branches and figures b to the 'upper' branches of the curves $A(h)$, $h(\tau)$, and $A(\tau)$.

To conclude this section, we analyse the distribution of the pseudotemperature of the particulate phase in the boundary layer. Since the distribution of the particle concentration in the boundary layer is given by the solution of Eq. (41) (for intermediate concentrations at the wall, α_+, the explicit formula (46) can be used), from (34), (52), (56), (62) and (63) we find the pseudotemperature in the boundary layer in the form

$$E^* = \frac{\beta^{2/3} G A(\tau) \Psi^{1/6}(0)}{(18)^{1/3} \Phi^{1/3}(0) \Psi^{2/3}(\alpha_+)} \frac{S(\alpha_+)}{S\big(\alpha^*(x)\big)}. \tag{65}$$

The pseudotemperature at the wall, using (51), can now be calculated as

$$E_+(\beta, G, \alpha_+) = 0.707 \frac{\beta^{2/3} G A(\tau(\beta, G, \alpha_+))}{\Psi^{2/3}(\alpha_+)}. \tag{66}$$

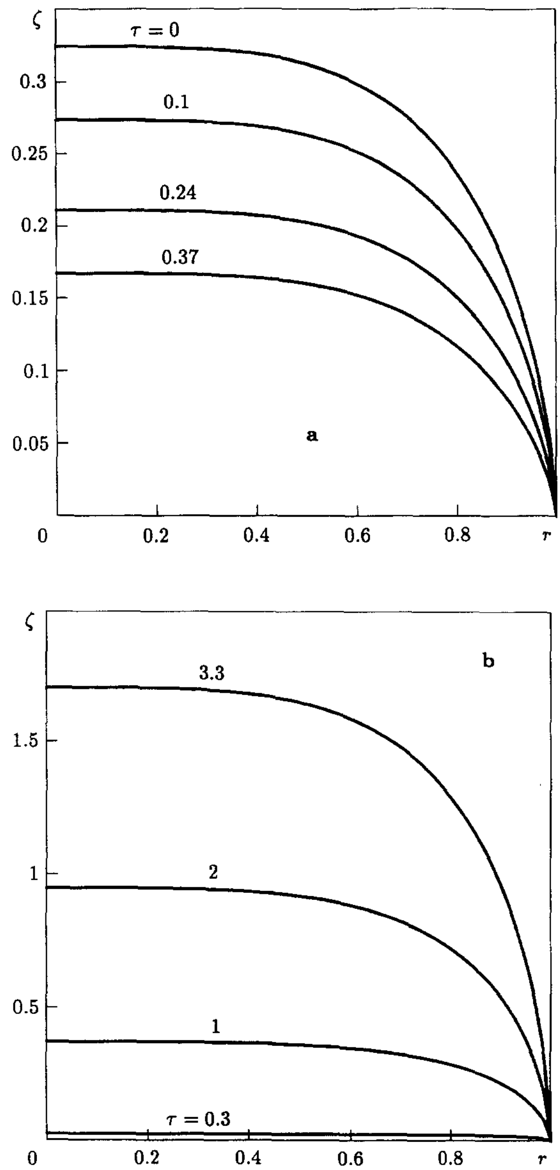

Fig. 4. Radial distribution of the scaled particle pseudotemperature in the bulk: (a) regime of upward particulate flow; (b) regime of downward particulate flow.

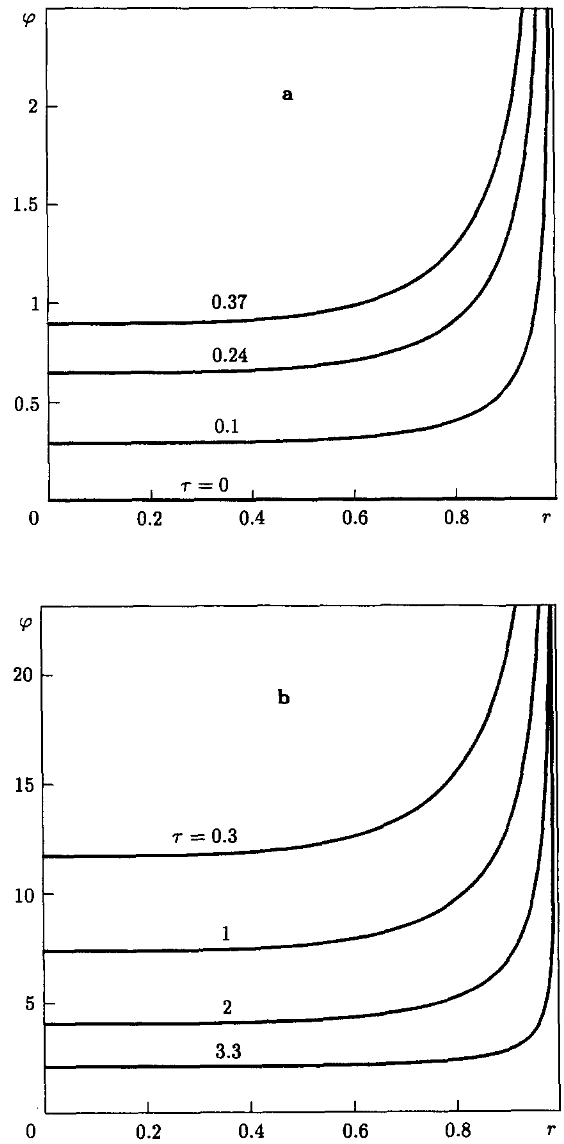

Fig. 5. Radial distribution of the scaled particle concentration in the bulk: (a) regime of upward particulate flow; (b) regime of downward particulate flow.

Since $G = O(\beta^{2/3})$, the order of magnitude of E_+ is $\beta^{4/3}$, and this gives $\nu = \frac{4}{3}$ in (33) (in accordance with the order of magnitude of the constant $C = O(\beta^{4/3})$ in (34) found above).

4. Velocity distribution: Interpretation of flow regimes

We start with the derivation of the equations governing the velocity distribution in the bulk and the boundary layer and with the estimate of the orders of magnitude of the velocity in both domains.

Since $\alpha_* = O(\beta^{2/3})$, we write the equation for the velocity in the outer region v_* in the form

$$\sqrt{\zeta}\,\frac{dv_*}{dr} = k_*\left\{-r + \frac{h(\tau)}{r}\int\limits_0^r \frac{r\,dr}{\zeta}\right\}, \tag{67}$$

where $\zeta = \zeta(r,\tau)$ is the solution of the problem (57), (60) and the coefficient k_* is

$$k_* = \frac{\beta^{1/3}\Gamma^{1/2}\Psi^{1/4}(0)}{2^{1/2}\Phi^{3/4}(0)} \simeq 3.239\,\beta^{1/3}\Gamma^{1/2}. \tag{68}$$

Eq. (67) is valid within the interval $0 \leqslant r < 1 - O(\beta)$. From (67) and (68) it follows that the order of magnitude of the particle velocity in the bulk is $\beta^{1/3}$.

To derive the equation for the velocity in the boundary layer, we first represent Eq. (67) in the asymptotic form

$$\frac{dv_*}{dr} = -k_*\frac{L(\tau)}{\sqrt{\zeta}} \quad \text{as } r \to 1\,. \tag{69}$$

Here

$$L(\tau) = 1 - h(\tau)\int\limits_0^1 \frac{r\,dr}{\zeta(r,\tau)} \tag{70}$$

which is represented in Fig. 6. It is important to note that the function $L(\tau)$ also has two branches; the 'upper' one corresponds to the 'upper' branches of the functions $h(\tau)$, $A(\tau)$ and $A(h)$; for this branch $L(\tau) > 0$ for all τ from the interval $0 < \tau < \tau_c$. For the 'lower' branch of $L(\tau)$, corresponding to the 'lower' branch of $h(\tau)$, $L(\tau) < 0$ for all τ.

Since $\zeta = A(\tau)(1-r)^{2/3}$ as $r \to 1$, from (67)–(70) it follows the order of magnitude of the velocity v^* in the boundary layer:

$$v^* = O(\beta)\,. \tag{71}$$

Since the integral term in Eq. (28) can be represented in the form

$$\frac{1}{r}\int\limits_0^r \alpha r\,dr = \int\limits_0^r \alpha_* r\,dr + O(\beta) = \frac{1}{2}\beta^{2/3}\Gamma h(\tau)\int\limits_0^1 \frac{r\,dr}{\zeta(r,\tau)} + O(\beta) \tag{72}$$

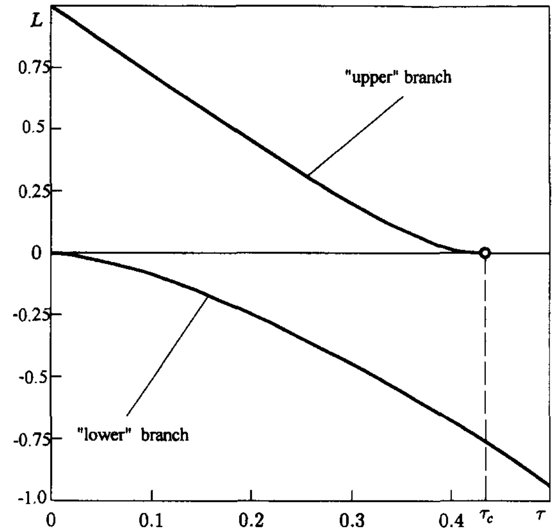

Fig. 6. Function $L(\tau)$ ('upper' branch, $L(\tau) > 0$, corresponds to the regime of upward particulate flow).

as $1 - O(\beta) \leqslant r \leqslant 1$, the equation for the leading asymptotic term of the velocity in the boundary layer immediately follows from (28) in the form

$$\Phi(\alpha^*)\sqrt{E^*}\, dv^*/dx = \tfrac{1}{2}\beta^{5/3}\Gamma L(\tau)\,. \tag{73}$$

This equation must be considered subject to the first boundary condition in (22).

The particle velocity distribution in the bulk can be found irrespective of the velocity distribution in the boundary layer. Indeed, from (71) it follows that Eq. (67) can be solved within the whole interval $0 \leqslant r \leqslant 1$ subject to the first boundary condition in (22).

Using the obtained numerical solution for $\zeta(r,\tau)$ to calculate the RHS of (67), we find that, for the 'upper' branch of the curve $h(\tau)$,

$$\frac{h(\tau)}{r}\int_0^r \frac{r\,dr}{\zeta(r,\tau)} < 1 \tag{74}$$

for all τ in the interval $0 < \tau \leqslant \tau_c$. The opposite inequality holds for the 'lower' branch. It then follows that the solution corresponding to the 'lower' branch of the curves $A(h)$, $h(\tau)$, $A(\tau)$ and $L(\tau)$ (see Figs. 1–3 and 6) describes the regime of downward flow of particles in the entire volume of the tube. Since the inequality (74) corresponds to the regime of upward flow in the bulk, using (63) and the found above value $\tau_c \simeq 0.43$, the necessary condition for the existence of the upward flow of solid particles can be found in the form

$$G > 3.55\,\beta^{2/3}\Psi^{1/6}(0)S(\alpha_+)\Psi^{-2/3}(\alpha_+)\,. \tag{75}$$

Condition (75) actually gives a minimum gas pressure gradient sufficient to provide an upward flow of solid particles. The following interpretation can be given to the phenomenon of existence of two different solutions for the same value of $\tau < \tau_c$. For this it is convenient to consider small τ when the existence of two solutions is guaranteed. Small values of τ may correspond to a relatively high gas pressure gradient, when the regime of upward flow of particles must be expected, as well as to the case of a low pressure gradient and very low $\beta \sim D_p/R$, such that the tube radius is extremely large compared with the particle size. When R is very large the flow in the bulk is practically not affected by the wall region and the low pressure gradient is not sufficient to build up an upward flow of the particulate phase in the bulk.

The solution of Eq. (67) subject to the first boundary condition in (22) can be represented in the explicit form to give the following particle velocity distribution in the bulk:

$$v_* = k_*[H(r,\tau) - h(\tau)I(r,\tau)]\,, \tag{76}$$

where

$$H(r,\tau) = \int\limits_r^1 \frac{r'\,dr'}{\sqrt{\zeta(r',\tau)}}\,, \tag{77a}$$

$$I(r,\tau) = \int\limits_r^1 \frac{1}{r'\sqrt{\zeta(r',\tau)}} \left(\int\limits_0^{r'} \frac{r''\,dr''}{\zeta(r'',\tau)} \right) dr'\,. \tag{77b}$$

The velocity distribution obtained by numerical integration of (77) for various τ corresponding to the regime of upward particulate flow (i.e. for the 'upper' branch of $h(\tau)$, $\tau \leqslant \tau_c$) is depicted in Fig. 7.

To conclude this section, we note that the details of the velocity distribution in the boundary layer are not significant for the present analysis (all essential physical information is given by the analysis of the particle velocity in the bulk). However, below we give the results of calculation of the velocity in the boundary layer for the special case of low particle concentration at the wall. In this case $\Psi(\alpha_+)$ can be replaced by $\Psi(0)$, and $S(\alpha_+) \simeq \alpha_+$, so that, using the approximation (46) for the particle concentration distribution in the boundary layer, from (65) we find the pseudotemperature in the inner region:

$$E^* = \frac{\beta^{4/3}\Gamma A(\tau)}{(18)^{1/3}\Phi^{1/3}(0)\Psi^{1/2}(0)}\,(1+\tfrac{2}{3}x)^{2/3}\,. \tag{78}$$

Integrating Eq. (72) we find the required velocity distribution for small α_+:

$$v^* = 10.17\,\beta\Gamma^{1/2}L(\tau)A^{-1/2}(\tau)\left[(1+\tfrac{2}{3}x)^{2/3} - 1\right]. \tag{79}$$

Here we should make an important remark. Since $L(\tau) > 0$ in the regime of upward particulate flow (for $\tau < \tau_c$), the downward flow in the wall region never occurs at any values of the macroscopic parameters, and the radial particle velocity distribution remains monotonic from the centre to the wall, i.e. for $0 \leqslant r \leqslant 1$, no matter how low

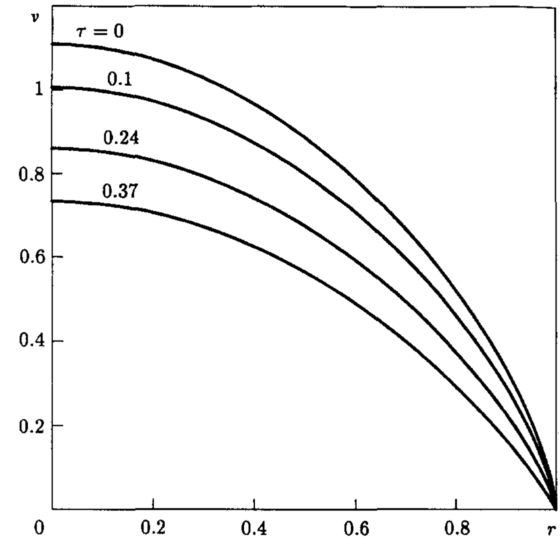

Fig. 7. Dimensionless particle-phase velocity distribution in the bulk in the regime of upward particulate flow.

the gas pressure gradient is (provided the latter satisfies the condition (75)). However, the numerous experimental data for upward granular flow in vertical tubes show the formation of the downward particulate flow in the near-wall region at sufficiently low pressure gradient. It can be expected that an absence of the downward flow in the near-wall region was caused in the present theory by the use of the boundary condition for the particle velocity in the approximate form $v = 0$ at $r = 1$. Indeed, such a condition (used also in the numerical analysis [3,5]) has been formulated only in a sense that the particle velocity at the wall is much lower than that in the bulk, as follows, for example, from [9]. Such an approximate condition is not sufficient to describe a possible downward flow in the near-wall region, and more detailed condition derived in [9] is needed to be included in the theory for this purpose.

5. Integrated parameters and particle concentration at the wall

The integrated parameters are the total mass fluxes of the gas and particle phase. As soon as the total particle flux has been found as a function of the given macroscopic parameters, $J = J(\beta, G, \alpha_+)$, the particle concentration can be calculated as $\alpha_+ = \alpha_+(\beta, G, J)$ and the problem solved in its original formulation (i.e. for the given J).

The dimensional integral flux of particles can be conveniently represented through the dimensionless integral flux j as follows:

$$J = 2\pi\rho R^3\sqrt{g/D_{\mathrm{p}}}\,j\,, \tag{80}$$

where

$$j = \int_0^1 \alpha v r \, dr \,. \tag{81}$$

The integral (81) can be represented as the sum of two integrals, one taken over the bulk and the other over the boundary layer. Since $\alpha = O(\beta^{2/3})$ and $v = O(\beta^{1/3})$ in the bulk, the contribution of the first integral is of the order β. In the boundary layer $\alpha = \alpha^* = O(1)$ and $v^* = O(\beta)$; since the thickness of the boundary layer is $O(\beta)$, the contribution of the second integral is $O(\beta^2)$, so that it can be neglected compared with the first one. To calculate the leading asymptotic term of j with respect to β, the integration of the first integral can be then carried over the whole interval $0 \leqslant r \leqslant 1$, so that for the dimensionless flux we obtain

$$j = \int_0^1 \alpha_* v_* r \, dr \,. \tag{82}$$

Substituting the first relation in (52), the relation (62), the definition of the parameter τ (63), and (76)–(77) together with (68) into (82), and using the definition of the parameter β given by (24), we obtain from (80) the following result for the dimensional integral flux of solid particles:

$$J = k_j R^{7/3} D_{\mathrm{p}}^{1/6} \sqrt{\rho |\Pi|}\, B(\alpha_+) W(\tau) \,, \tag{83}$$

where the coefficient k_j is the constant value defined through $\Phi(0)$ and $\Psi(0)$,

$$k_j = 2^{5/6} 3^{-2/3} \pi \Phi^{-3/4}(0) \Psi^{5/6}(0) \simeq 13.01 \,, \tag{84}$$

and the functions $B(\alpha_+)$ and $W(\tau)$ are as follows:

$$B(\alpha_+) = \frac{S(\alpha_+)}{[w\alpha_+ g_0(\alpha_+)]^{2/3}} \,, \tag{85}$$

$$W(\tau) = A(\tau) \left\{ \int_0^1 \frac{H(r,\tau) r \, dr}{\zeta(r,\tau)} - h(\tau) \int_0^1 \frac{I(r,\tau) r \, dr}{\zeta(r,\tau)} \right\} . \tag{86}$$

Here the functions $H(r,\tau)$ and $J(r,\tau)$ are given by (77). From (9) and (8b) it follows that

$$B(\alpha_+) = w^{-2/3} \alpha_+^{1/3} \tag{87}$$

for not very high α_+ (actually for $\alpha_+ \leqslant 0.15$, which is a realistic estimate for granular flow in vertical tubes). The function $W(\tau)$ is shown in Fig. 8 for the 'upper' branch corresponding to the regime of upward flow of solid particles. It is seen from Fig. 8

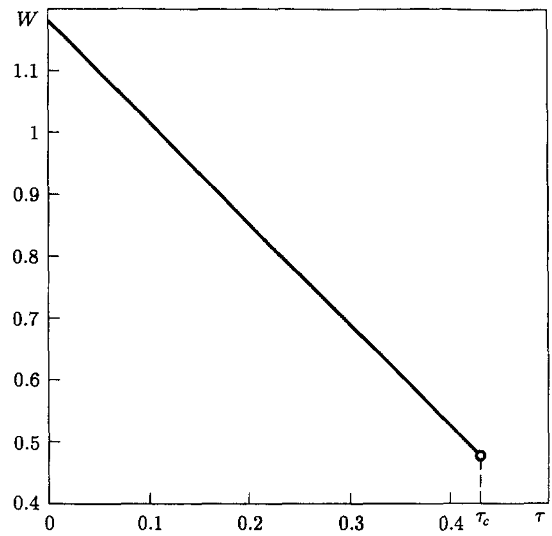

Fig. 8. Function $W(\tau)$ for the regime of upward particulate flow.

that the function $W(\tau)$ within the interval $0 \leqslant \tau \leqslant \tau_c \simeq 0.43$ can be approximated by the linear relation

$$W(\tau) \simeq 0.485 - 1.619(\tau_c - \tau). \tag{88}$$

The results given by (83)–(86) determine the particle concentration at the wall α_+ as a function of the total mass flux J and other macroscopic parameters of the system, and, therefore, complete the solution of the problem in the original formulation.

The results obtained for the integral flux of particles can be represented in a simple explicit form in the practically important case of small τ. From the definition of τ (63) and the properties of the function $S(\alpha)$ at small α it follows that small values of τ may correspond to relatively high pressure gradients as well as to the case of low α_+ and not very high $G \sim \beta^{2/3}$. To illustrate the latter case let us suppose that $G = 2\beta^{2/3}$; in this case the parameter τ is small already with $\alpha_+ \simeq 0.2$ corresponding to the range of parameters of practical interest. Since $h(\tau) \to 0$ and $A(\tau) \to A_0 \simeq 0.969$ as $\tau \to 0$, the value $W(\tau)$ in (83) becomes

$$W(0) = A_0 \int_0^1 \frac{H(r,0) r\, dr}{\zeta(r,0)} \simeq 1.181. \tag{89}$$

Substituting (88) and (86) into (83) we obtain the following explicit expression for the integral flux of the particulate phase valid for not very high α_+:

$$J = 15.36 R^{7/3} D_p^{1/6} w^{-2/3} \alpha_+^{1/3} \sqrt{\rho |\Pi|}. \tag{90}$$

The latter formula gives an explicit dependence of α_+ on J and other macroscopic parameters (i.e., R, D_p, ρ, Π and w). It must be noted that the integral flux of solid particles depends strongly on the tube radius (as $R^{7/3}$), but very weakly on the particle size. We should also mention here that particle–wall collisions are usually very inelastic so that $e_w = 0$ in (15) and $w = 2.09$ in (90) can be assumed.

To conclude this section we also calculate the dimensional integral mass flux of the gas phase. In case of sufficiently small particles the viscous drag coefficient b in (1) and (2) is very large, so that the terminal velocity of particles is much lower than the mean velocity of particles in the bulk and, therefore, the gas velocity practically coincides with the velocity of solid particles. It then follows that the dimensional integral gas flux can be calculated in the form

$$J_g = 2\pi\rho_g R^3 \sqrt{g/D_p}\, j_g\,, \tag{91}$$

where

$$j_g = \int_0^1 \varepsilon v r\, dr\,. \tag{92}$$

Applying the procedure used for the derivation of the leading asymptotic term of the particle flux (82) and recalling that $\alpha = O(\beta^{2/3})$ in the bulk, we arrive to the following expression for the leading asymptotic term of the dimensionless gas flux:

$$j_g = \int_0^1 v_*(r,\tau) r\, dr\,, \tag{93}$$

where v_* is the particle velocity in the bulk. By means of (76), (77) and (68) we find the dimensional gas flux from (91) and (93) in the form

$$J_g = 20.35\, \rho_g R^3 \sqrt{\frac{|\Pi|}{\rho D_p}}\, W_g(\tau)\,, \tag{94}$$

where the function $W_g(\tau)$, given by the relation

$$W_g(\tau) = \int_0^1 H(r,\tau) r\, dr - h(\tau) \int_0^1 I(r,\tau) r\, dr \tag{95}$$

is shown in Fig. 9 for the 'upper' branch of $h(\tau)$ corresponding to the regime of upward particulate flow. It must be noted that the integral gas flux, unlike the flux of solid particles, is strongly affected by the particle size (as $D_p^{-1/2}$). For the case of small τ we find the following simple approximate result for J_g:

$$J_g = 6.48\rho_g R^3 \sqrt{|\Pi|/(\rho D_p)}\,. \tag{96}$$

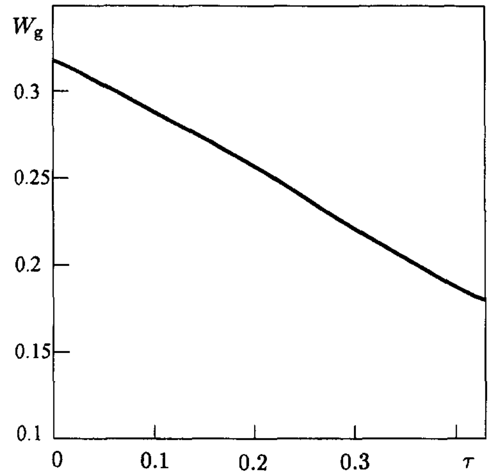

Fig. 9. Function $W_g(\tau)$ for the regime of upward particulate flow.

6. Conclusion

In this work, based on the equations of the two-fluid model together with the boundary conditions and constitutive relations following from the collision version of the kinetic theory of a suspension of solid particles in a gas [2], we developed the approximate asymptotic theory of the developed gas-particulate flow through vertical tubes. This problem has been earlier treated numerically in [3–5]. Since the mathematical formulation of the problem shows a singular behaviour of the flow parameters at the wall, the problem can be treated analytically by means of the method of matched asymptotic expansions. The solution obtained clearly shows the existence of two main regions in the flow: the dense particle-phase boundary layer and the dilute bulk, is consistent with the numerical results [3–4], that gives an explicit explanation to the very well known phenomenon of the formation of the dense layer of particles at the wall. The radial distributions of particle phase concentration and velocity are found, and the integrated parameters (total mass fluxes) are calculated as functions of macroscopic parameters of the system. The condition for the existence of the upward particulate flow (i.e. the critical pressure gradient) is derived.

It is important to underline here that the theory is developed for intermediate values of the gas pressure gradient. The earlier development [7] of the analytical model assumed very high gas pressure gradients, while for the pressure gradients lower than these considered in the present work only a downward flow of particles exists, which is of much less theoretical and practical interest. We can, therefore, expect the results of the present work to cover the theoretically and practically important range of parameters of two-phase flow.

One more concluding remark is to be made here. To develop the above theory, the equations, boundary conditions and constitutive relations were used in the form following from the kinetic theory of a suspension of solid particles in a gas based on the assumption on the direct (generally inelastic) collisions between particles as the only mechanism of interparticle interaction. It can be, however, often expected that the general structure of basic equations, constitutive relations and boundary conditions remains the same in case of more general assumptions on the character of interparticle interaction (e.g. particle–particle and particle–wall interactions through the gas in a two-phase turbulent flow, see, for example, [11]). It is then hoped that the method developed in the present work can be as well applied for an analytical study of more complicated models of gas-particulate two-phase flow.

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