

Comment on: "A novel approach for solving the Fisher equation using Exp-function method" [Phys. Lett. A 372 (2008) 3836-3840]

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Abstract

Using Exp-function method Öziş and Köroğlu [Öziş T, Köroğlu C., Phys. Lett. A 372 (2008) 3836 - 3840] have found exact "solutions" of the Fisher equation. In this comment we demonstrate that all these solutions do not satisfy the Fisher equation. The efficiency of application of Exp-function method to search for exact solutions of nonlinear differential equations is questioned by us.

Key words:

Nonlinear evolution equation; Fisher equation; Exact solution; Exp-function method; Simplest equation method

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In [1] Öziş and Köroğlu used the Exp-function method [2] to look for exact solutions of the Fisher equation

$$u_t - u_{xx} = u(1 - u). \quad (0.1)$$

Eq. (0.1) was studied by Kolmogorov, Petrovskii and Piskunov [3] and by Fisher [4].

Taking the travelling wave $u(x, t) = U(\eta)$, $\eta = kx + wt$ into account authors [1] have presented Eq. (0.1) in the form

$$k^2 U_{\eta\eta} + cw U_{\eta} + U - U^2 = 0, \quad (0.2)$$

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where c , w and k are parameters of Eq. (0.2). Solutions of Eq. (0.1) were first found by Ablowitz and Zepetella in [5]. Later solutions of Eq. (0.1) were found many times [6, 7].

Using the Exp-function method Öziş and Köroğlu [1] found three "solutions" of Eq. (0.2). These "solutions" were given in the form

$$U^{(1)} = \frac{(1 - 2k^2)b_{-1}}{b_0 \exp(\eta) + b_{-1}}, \quad \eta = kx + wt, \quad (0.3)$$

$$U^{(2)} = \frac{(1 - 8k^2)b_{-1}}{b_1 \exp(2\eta) + b_{-1}}, \quad \eta = kx + wt, \quad (0.4)$$

$$U^{(3)} = \frac{b_0 - 2k^2 b_{-1} \exp(-\eta)}{b_0 + b_{-1} \exp(-\eta)}, \quad \eta = kx + wt. \quad (0.5)$$

However all these "solutions" do not satisfy equation (0.1). We can note this fact without substitutions solutions (0.3) - (0.5) into Eq. (0.1). The matter is solution of Eq. (0.1) has the pole of the second order but all functions (0.3) - (0.5) with poles of the first order.

To be on the save side we have substituted functions (0.3), (0.4) and (0.5) into Eq. (0.2) and have obtained after multiplying on $(b_0 \exp(\eta) + b_{-1})^3$, $(b_1 \exp(2\eta) + b_{-1})^3$ and $(b_0 + b_{-1} \exp(-\eta))^3$ the following expressions

$$E^{(1)} = (1 - 2k^2) [2k^2 b_{-1}^2 + b_0 (1 + k^2 - cw) (b_{-1} e^\eta + b_0 e^{2\eta})], \quad (0.6)$$

$$E^{(2)} = (1 - 8k^2) [8k^2 b_{-1}^2 + b_1 (1 + 4k^2 - 2cw) (b_{-1} e^{2\eta} + b_1 e^{4\eta})], \quad (0.7)$$

$$E^{(3)} = b_0 b_{-1} (1 + k^2) (1 + cw - k^2) (b_0 + b_{-1} e^{-\eta}) - 2b_{-1}^3 k^2 (2k^2 + 1) e^{-2\eta}. \quad (0.8)$$

Taking into account $w = \frac{1+k^2}{c}$ in (0.6), $w = \frac{1+4k^2}{2c}$ in (0.7) and $w = \frac{k^2-1}{c}$ in (0.8) we can simplify these expressions but these ones are not equal to zero in the general case and we can see that functions (0.3) - (0.5) do not satisfy Eq. (0.1).

Solitary wave solutions of Eq. (0.1) can be found using the singular manifold method [8–10], the tanh-function method [11, 12], the simplest equation method [13–16] and so on.

Without loss of generality let us apply the simplest equation method [15, 16] to search for exact solutions of the Fisher equation in the form

$$U_{\eta\eta} + cU_\eta + U - U^2 = 0, \quad (0.9)$$

We assume

$$U = m_0 + m_1 Y + m_2 Y^2, \quad Y \equiv Y(\eta), \quad (0.10)$$

where m_0, m_1, m_2 are constants and $Y(\eta)$ satisfies the Riccati equation in the form

$$Y_\eta = -Y^2 + \beta. \quad (0.11)$$

Taking into consideration the transformation

$$Y = \frac{\psi_\eta}{\psi}, \quad (0.12)$$

we can present formula (0.10) in the form

$$v = m_0 + m_1 \frac{\psi_\eta}{\psi} + m_2 \left(\frac{\psi_\eta}{\psi} \right)^2. \quad (0.13)$$

As this takes place Eq. (0.11) can be written in the form of the second-order linear equation

$$\psi_{\eta\eta} - \beta \psi = 0. \quad (0.14)$$

Substituting (0.10) into (0.9) and equating to zero expressions at different degrees of function $Y(z)$ we obtain the system of the algebraic solutions with respect to coefficients m_0, m_1, m_2 and parameter β .

We can also apply formulae (0.13) and (0.14) for finding exact solutions of nonlinear differential equations. From comparison formulae (0.10) - (0.14) we can see this is the same approach [16].

Solving the set of the algebraic equations with respect to coefficients m_0, m_1 and m_2 we obtain in (0.13)

$$m_2 = 6 \quad m_1 = -\frac{6}{5}c, \quad m_0 = \frac{1}{2} - 4\beta - \frac{c^2}{50}, \quad \beta = \frac{c^2}{100} \quad (0.15)$$

$$c_{1,2} = \pm \frac{5i\sqrt{6}}{6}, \quad c_{3,4} = \pm \frac{5\sqrt{6}}{6}. \quad (0.16)$$

Solutions take the form

$$U = \frac{1}{2} - 4\beta - \frac{c^2}{50} - \frac{6c\psi_\eta}{5\psi} + \frac{6\psi_\eta^2}{\psi^2}, \quad (0.17)$$

where

$$\psi(\eta) = C_1 e^{\eta\sqrt{\beta}} + C_2 e^{-\eta\sqrt{\beta}}, \quad (0.18)$$

where C_1 and C_2 are arbitrary constants. Taking into account (0.15), (0.16) and (0.18) we have four exact solutions of (0.9) in the form

$$U_{1,2} = \frac{3}{4} \pm \frac{i}{2} \tan \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\} + \frac{1}{4} \tan^2 \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\}, \quad (0.19)$$

$$U_{3,4} = \frac{1}{4} \left(1 \mp \tanh \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\} \right)^2, \quad (0.20)$$

where η_0 is arbitrary constant. Substituting (0.19) and (0.20) into Eq. (0.9) at $c = c_{(1,2)}$ and at $c = c_{(3,4)}$ we can convince that (0.19) and (0.20) are solutions of Eq. (0.9).

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