Comment on: "A novel approach for solving the Fisher equation using Exp-function method"[Phys. Lett. A 372 (2008) 3836-3840]

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Abstract

Using Exp-function method Öziş and Köroğlu [Öziş T, Köroğlu C., Phys. Lett. A 372 (2008) 3836 - 3840] have found exact "solutions" of the Fisher equation. In this comment we demonstrate that all these solutions do not satisfy the Fisher equation. The efficiency of application of Exp-function method to search for exact solutions of nonlinear differential equations is questioned by us.

Key words:
Nonlinear evolution equation; Fisher equation; Exact solution; Exp-function method; Simplest equation method
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In [1] Öziş and Köroğlu used the Exp-function method [2] to look for exact solutions of the Fisher equation

$$u_t - u_{xx} = u\,(1-u). \tag{0.1}$$

Eq. (0.1) was studied by Kolmogorov, Petrovskii and Piskunov [3] and by Fisher [4].

Taking the travelling wave $u(x,t) = U(\eta)$, $\eta = k\,x + w\,t$ into account authors [1] have presented Eq. (0.1) in the form

$$k^2\,U_{\eta\eta} + c\,w\,U_\eta + U - U^2 = 0, \tag{0.2}$$

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where c, w and k are parameters of Eq. (0.2). Solutions of Eq. (0.1) were first found by Ablowitz and Zepetella in [5]. Later solutions of Eq. (0.1) were found many times [6, 7].

Using the Exp-function method Öziş and Köroğlu [1] found three "solutions" of Eq. (0.2). These "solutions" were given in the form

$$U^{(1)} = \frac{(1-2\,k^2)\,b_{-1}}{b_0\,\exp{(\eta)}+b_{-1}}, \qquad \eta = k\,x + w\,t, \tag{0.3}$$

$$U^{(2)} = \frac{(1-8\,k^2)\,b_{-1}}{b_1\,\exp{(2\,\eta)}+b_{-1}}, \qquad \eta = k\,x + w\,t, \tag{0.4}$$

$$U^{(3)} = \frac{b_0 - 2\,k^2\,b_{-1}\,\exp{(-\eta)}}{b_0 + b_{-1}\exp{(-\eta)}}, \qquad \eta = k\,x + w\,t. \tag{0.5}$$

However all these "solutions" do not satisfy equation (0.1). We can note this fact without substitutions solutions (0.3) - (0.5) into Eq. (0.1). The matter is solution of Eq. (0.1) has the pole of the second order but all functions (0.3) - (0.5) with poles of the first order.

To be on the save side we have substituted functions (0.3), (0.4) and (0.5) into Eq. (0.2) and have obtained after multiplying on $(b_0\,\exp{(\eta)}+b_{-1})^3$, $(b_1\,\exp{(2\,\eta)}+b_{-1})^3$ and $(b_0+b_{-1}\exp{(-\eta)})^3$ the following expressions

$$E^{(1)} = (1-2\,k^2)\left[2\,k^2{b_{-1}}^2 + b_0\left(1+k^2-c\,w\right)\left(b_{-1}e^{\eta}+\,b_0\,e^{2\,\eta}\right)\right], \tag{0.6}$$

$$E^{(2)} = \left(1-8k^2\right)\left[8k^2{b_{-1}}^2 + b_1\left(1+4k^2-2cw\right)\left(b_{-1}e^{2\eta}+b_1\,e^{4\eta}\right)\right], \tag{0.7}$$

$$E^{(3)} = b_0 b_{-1}\,(1+k^2)\,(1+c\,w-k^2)(b_0+b_{-1}\,e^{-\eta}) - 2\,{b_{-1}}^3k^2\left(2\,k^2+1\right)e^{-2\,\eta}. \tag{0.8}$$

Taking into account $w=\frac{1+k^2}{c}$ in (0.6), $w=\frac{1+4\,k^2}{2\,c}$ in (0.7) and $w=\frac{k^2-1}{c}$ in (0.8) we can simplify these expressions but these ones are not equal to zero in the general case and we can see that functions (0.3) - (0.5) do not satisfy Eq. (0.1).

Solitary wave solutions of Eq. (0.1) can be found using the singular manifold method [8–10], the tanh-function method [11, 12], the simplest equation method [13–16] and so on.

Without loss of generality let us apply the simplest equation method [15, 16] to search for exact solutions of the Fisher equation in the form

$$U_{\eta\eta} + c\,U_{\eta} + U - U^2 = 0, \tag{0.9}$$

We assume

$$U = m_0 + m_1\,Y + m_2\,Y^2, \qquad Y \equiv Y(\eta), \tag{0.10}$$

where m_0, m_1, m_2 are constats and $Y(\eta)$ satisfies the Riccati equation in the form

$$Y_\eta = -Y^2 + \beta. \tag{0.11}$$

Taking into consideration the transformation

$$Y = \frac{\psi_\eta}{\psi}, \tag{0.12}$$

we can present formula (0.10) in the form

$$v = m_0 + m_1\,\frac{\psi_\eta}{\psi} + m_2\,\left(\frac{\psi_\eta}{\psi}\right)^2. \tag{0.13}$$

As this takes place Eq. (0.11) can be written in the form of the second-order linear equation

$$\psi_{\eta\eta} - \beta\,\psi = 0. \tag{0.14}$$

Substituting (0.10) into (0.9) and equating to zero expressions at different degrees of function $Y(z)$ we obtain the system of the algebraic solutions with respect to coefficients m_0, m_1, m_2 and parameter β.

We can also apply formulae (0.13) and (0.14) for finding exact solutions of nonlinear differential equations. From comparison formulae (0.10) - (0.14) we can see this is the same approach [16].

Solving the set of the algebraic equations with respect to coefficients m_0, m_1 and m_2 we obtain in (0.13)

$$m_2 = 6 \qquad m_1 = -\frac{6}{5}\,c, \qquad m_0 = \frac{1}{2} - 4\,\beta - \frac{c^2}{50}, \qquad \beta = \frac{c^2}{100} \tag{0.15}$$

$$c_{1,2} = \pm\,\frac{5\,i\sqrt{6}}{6}, \qquad c_{3,4} = \pm\,\frac{5\,\sqrt{6}}{6}. \tag{0.16}$$

Solutions take the form

$$U = \frac{1}{2} - 4\,\beta - \frac{c^2}{50} - \frac{6\,c\,\psi_\eta}{5\,\psi} + \frac{6\,\psi_\eta^2}{\psi^2}, \tag{0.17}$$

where

$$\psi(\eta) = C_1\,e^{\eta\,\sqrt{\beta}} + C_2\,e^{-\eta\,\sqrt{\beta}}, \tag{0.18}$$

where C_1 and C_2 are arbitrary constants. Taking into account (0.15), (0.16) and (0.18) we have four exact solutions of (0.9) in the form

$$U_{1,2} = \frac{3}{4} \pm \frac{i}{2} \tan\left\{\frac{\sqrt{6}\,(\eta - \eta_0)}{12}\right\} + \frac{1}{4} \tan^2\left\{\frac{\sqrt{6}\,(\eta - \eta_0)}{12}\right\}, \tag{0.19}$$

$$U_{3,4} = \frac{1}{4}\left(1 \mp \tanh\left\{\frac{\sqrt{6}\,(\eta - \eta_0)}{12}\right\}\right)^2, \tag{0.20}$$

where η_0 is arbitrary constant. Substituting (0.19) and (0.20) into Eq. (0.9) at $c = c_{(1,2)}$ and at $c = c_{(3,4)}$ we can convince that (0.19) and (0.20) are solutions of Eq. (0.9).

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