

A new note on exact complex travelling wave solutions for (2+1) – dimensional B – type Kadomtsev – Petviashvili equation

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Abstract

Exact solutions of the (2+1) – dimensional Kadomtsev – Petviashvili by Zhang [Zhang H., Applied Mathematics and Computation 216 (2010) 2771 – 2777] are considered. To look for "new types of exact solutions travelling wave solutions" of equation Zhang has used the G'/G – expansion method. We demonstrate that there is the general solution for the reduction by Zhang from the (2+1) – dimensional Kadomtsev – Petviashvili equation and all solutions by Zhang are found as partial cases from the general solution.

In recent paper [1] author has looked for exact solutions of the system of equations

$$\begin{aligned}u_y &= q_x \\v_x &= q_y \\q_t &= q_{xxx} + q_{yyy} + 6(qu)_x + 6(qv)_y\end{aligned}\tag{1}$$

where $u \equiv u(x, y, t)$, $v \equiv v(x, y, t)$ and $q \equiv q(x, y, t)$ are dependent variables with respect to x , y and t .

Zhang [1] looked for exact solutions of Eq.(1) using the travelling wave solutions taking into account the following variables

$$\begin{aligned}q(x, y, t) &= q(z), \quad u(x, y, t) = u(z), \quad v(x, y, t) = v(z), \\z &= i(\alpha x + \beta y + ct).\end{aligned}\tag{2}$$

As result he obtained the system of equations in the form

$$\begin{aligned}\beta u' - \alpha q' &= 0, \\ \alpha v' - \beta q' &= 0, \\ (\alpha^3 + \beta^3) q''' + c q' - 6\alpha(qu)' - 6\beta(qv)' &= 0.\end{aligned}\tag{3}$$

where $()'$ is derivative $\frac{d}{dz}$. Author [1] looked for solutions of system of equations using the the G'/G – expansion method and believes that he obtained "new types of exact complex travelling wave solutions of the (2+1) – dimensional BKP equation". He presented fourth pages of the journal with useless formulae which are very dangerous for young scientists.

Let us demonstrate that the system of equations (3) can be reduced to the nonlinear differential equation with exact solution in the form of the Weierstrass function. First of all let us note that we can integrate all equations (3) with respect to z . As result we have the following equations

$$u = \frac{\alpha}{\beta} q + C_1, \quad (4)$$

$$v = \frac{\beta}{\alpha} q + C_2, \quad (5)$$

$$(\alpha^3 + \beta^3) q'' + c q - 6 \alpha (q u) - 6 \beta (q v) = C_3, \quad (6)$$

where C_1 , C_2 and C_3 are arbitrary constants.

Substituting u and v from (4) and (5) into (6) we obtain the nonlinear second order differential equation in the form

$$q'' - \frac{6}{\alpha\beta} q^2 - C_4 q - C_5 = 0, \quad (7)$$

where

$$C_4 = \frac{6\alpha C_1 + 6\beta C_2 - c}{\alpha^3 + \beta^3}, \quad C_5 = \frac{C_3}{\alpha^3 + \beta^3}. \quad (8)$$

Multiplying Eq. (7) on q' and integrating the equation with respect to z we obtain the equation

$$(q')^2 - \frac{4}{\alpha\beta} q^3 - C_4 q^2 - 2C_5 q - C_6 = 0. \quad (9)$$

where C_4 , C_5 and C_6 are arbitrary constants. Solution of Eq.(9) is well known. Eq.(9) was solved more than one century ago [2, 3].

Using transformation

$$q = \alpha\beta\wp(z) - \frac{C_4\alpha\beta}{12}, \quad (10)$$

(where $\wp(z, g_2, g_3)$ is the Weierstrass function with invariants g_2 and g_3) into Eq. (9) we obtain the equation for the Weierstrass function

$$(\wp')^2 = 4\wp^3 - g_2\wp - g_3, \quad (11)$$

where invariants g_2 and g_3 take the form

$$g_2 = \frac{C_4^2}{12} - \frac{C_5}{\alpha\beta}, \quad (12)$$

$$g_3 = \frac{C_5 C_4}{12\alpha\beta} - \frac{C_4^3}{216} - \frac{C_6}{\alpha^2\beta^2}. \quad (13)$$

We can see that the general solution of Eq.(9) is expressed via the Weierstrass elliptic function [2]. As for the rational, periodic and solitary wave solutions of (9) we can find them as partial case from solution (10).

It is well known that the Weierstrass elliptic function can be presented via the elliptic function Jacobi as well [3, 4]. Now let us present the general solution of Eq. (9) using the elliptic function Jacobi. Assuming

$$q = -\frac{\alpha\beta}{2} y, \quad \omega = C_4, \quad \varepsilon = \frac{4C_5}{\alpha\beta}, \quad \delta = -\frac{4C_6}{\alpha^2\beta^2},$$

we have equation in the form

$$y_z^2 + 2y^3 - \omega y^2 + 2\varepsilon y + 2\delta = 0, \quad (14)$$

Assuming that m_1, m_2 and m_3 ($m_1 \geq m_2 \geq m_3$) are roots of the algebraic equation

$$y^3 - \frac{1}{2}\omega y^2 + \varepsilon y + \delta = 0, \quad (15)$$

we can write Eq. (14) in the form

$$y_z^2 = -2(y - m_1)(y - m_2)(y - m_3). \quad (16)$$

The general solution of Eq. (14) is expressed via the Jacobi elliptic function

$$y(z) = m_2 + (m_1 - m_3)cn^2\left\{\sqrt{\frac{m_1 - m_3}{2}}z, S^2\right\}, \quad S^2 = \frac{m_1 - m_2}{m_1 - m_3}, \quad (17)$$

where $cn(z)$ is the elliptic cosine. The general solution of equation (17) were first found by Korteweg and de Vries [3].

Comparison of Eq. (15) and Eq. (16) allows us to find relations between the roots m_1, m_2, m_3 and the constants $\omega, \varepsilon, \delta$ in the form

$$\begin{aligned} m_1 m_2 m_3 &= -\delta, & m_1 m_2 + m_1 m_3 + m_2 m_3 &= \varepsilon, \\ m_1 + m_2 + m_3 &= \frac{\omega}{2}. \end{aligned} \quad (18)$$

We have obtained that there are solutions of the Eq.(1), which are expressed via general solution (17). Solitary wave solutions of Eq. (9) arise when Eq. (15) has two equal roots [3,4].

So Zhang [1] found partial cases of the well known equation with solution in the form of elliptic function. In fact this popular error was discussed in many papers [5–14]. However some authors do not take this error into account.

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