CONTENTS

Preface . . . . . **xv**

Authors . . . . . **xvi**

Annotation . . . . . **xvii**

Some Notation and Remarks . . . . . **xviii**

Part I. Linear Equations With Two Independent Variables 1

1. Equations Containing One Derivative **3**

2. Linear Equations of the Form $f(x,y)\frac{\partial w}{\partial x} + g(x,y)\frac{\partial w}{\partial y} = 0$ **5**

2.1. Preliminary Remarks 5
2.1.1. Solution Method 5
2.1.2. Cauchy Problem (Initial Value Problem) 6
2.1.3. Examples 6

2.2. Equations Containing Power-Law Functions 7
2.2.1. Coefficients of Equations Are Linear in x and y 7
2.2.2. Coefficients of Equations Are Quadratic in x and y 9
2.2.3. Coefficients of Equations Contain Integer Powers of x and y 12
2.2.4. Coefficients of Equations Contain Fractional Powers 13
2.2.5. Coefficients of Equations Contain Arbitrary Powers of x and y 14

2.3. Equations Containing Exponential Functions 24
2.3.1. Coefficients of Equations Contain Exponential Functions 24
2.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 25

2.4. Equations Containing Hyperbolic Functions 30
2.4.1. Coefficients of Equations Contain Hyperbolic Sine 30
2.4.2. Coefficients of Equations Contain Hyperbolic Cosine 30
2.4.3. Coefficients of Equations Contain Hyperbolic Tangent 31
2.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 32
2.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 33

2.5. Equations Containing Logarithmic Functions 34
2.5.1. Coefficients of Equations Contain Logarithmic Functions 34
2.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 34

2.6. Equations Containing Trigonometric Functions 37
2.6.1. Coefficients of Equations Contain Sine 37
2.6.2. Coefficients of Equations Contain Cosine 39
2.6.3. Coefficients of Equations Contain Tangent 40
2.6.4. Coefficients of Equations Contain Cotangent 42
2.6.5. Coefficients of Equations Contain Different Trigonometric Functions 43

2.7. Equations Containing Inverse Trigonometric Functions 45
2.7.1. Coefficients of Equations Contain Arcsine 45
2.7.2. Coefficients of Equations Contain Arccosine 46
2.7.3. Coefficients of Equations Contain Arctangent 48
2.7.4. Coefficients of Equations Contain Arccotangent 49

2.8. Equations Containing Arbitrary Functions of x 51
2.8.1. Equations Contain Arbitrary and Power-Law Functions 51
2.8.2. Equations Contain Arbitrary and Exponential Functions 53
2.8.3. Equations Contain Arbitrary and Hyperbolic Functions 54
2.8.4. Equations Contain Arbitrary and Logarithmic Functions 55
2.8.5. Equations Contain Arbitrary and Trigonometric Functions 55
2.8.6. Equations Contain Arbitrary Functions and Their Derivatives 56
2.9. Equations Containing Arbitrary Functions of Different Arguments 58
2.9.1. Equations Contain Arbitrary Functions of x and Arbitrary Functions of y 58
2.9.2. Equations Contain One Arbitrary Function of Complicated Argument 59
2.9.3. Equations Contain Several Arbitrary Functions 61

3. Linear Equations of the Form $f(x,y)\frac{\partial w}{\partial x} + g(x,y)\frac{\partial w}{\partial y} = h(x,y)$ 65

3.1. Preliminary Remarks 65
3.1.1. Solution Methods 65
3.1.2. Cauchy Problem 66
3.1.3. Examples 67
3.2. Equations Containing Power-Law Functions 68
3.2.1. Coefficients of Equations Are Linear in x and y 68
3.2.2. Coefficients of Equations Are Quadratic in x and y 69
3.2.3. Coefficients of Equations Contain Other Power-Law Functions 70
3.2.4. Coefficients of Equations Contain Arbitrary Powers of x and y 70
3.3. Equations Containing Exponential Functions 72
3.3.1. Coefficients of Equations Contain Exponential Functions 72
3.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 73
3.4. Equations Containing Hyperbolic Functions 74
3.4.1. Coefficients of Equations Contain Hyperbolic Sine 74
3.4.2. Coefficients of Equations Contain Hyperbolic Cosine 75
3.4.3. Coefficients of Equations Contain Hyperbolic Tangent 76
3.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 76
3.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 77
3.5. Equations Containing Logarithmic Functions 77
3.5.1. Coefficients of Equations Contain Logarithmic Functions 77
3.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 78
3.6. Equations Containing Trigonometric Functions 79
3.6.1. Coefficients of Equations Contain Sine 79
3.6.2. Coefficients of Equations Contain Cosine 79
3.6.3. Coefficients of Equations Contain Tangent 80
3.6.4. Coefficients of Equations Contain Cotangent 80
3.6.5. Coefficients of Equations Contain Different Trigonometric Functions 81
3.7. Equations Containing Inverse Trigonometric Functions 82
3.7.1. Coefficients of Equations Contain Arcsine 82
3.7.2. Coefficients of Equations Contain Arccosine 83
3.7.3. Coefficients of Equations Contain Arctangent 83
3.7.4. Coefficients of Equations Contain Arccotangent 84
3.8. Equations Containing Arbitrary Functions 85
3.8.1. Coefficients of Equations Contain Arbitrary Functions of x 85
3.8.2. Equations Contain Arbitrary Functions of x and Arbitrary Functions of y 87
3.8.3. Equations Contain Arbitrary Functions of Complicated Arguments 88
3.8.4. Equations Contain Arbitrary Functions of Two Variables 89

4. Linear Equations of the Form $f(x,y)\frac{\partial w}{\partial x} + g(x,y)\frac{\partial w}{\partial y} = h(x,y)w$ 91
4.1. Preliminary Remarks 91
4.1.1. Solution Methods 91
4.1.2. Examples 92
4.2. Equations Containing Power-Law Functions 93
4.2.1. Coefficients of Equations Are Linear in x and y 93
4.2.2. Coefficients of Equations Are Quadratic in x and y 94
4.2.3. Coefficients of Equations Contain Other Power-Law Functions 95
4.2.4. Coefficients of Equations Contain Arbitrary Powers of x and y 96
4.3. Equations Containing Exponential Functions 97
4.3.1. Coefficients of Equations Contain Exponential Functions 97
4.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 98
4.4. Equations Containing Hyperbolic Functions 99
4.4.1. Coefficients of Equations Contain Hyperbolic Sine 99
4.4.2. Coefficients of Equations Contain Hyperbolic Cosine 100
4.4.3. Coefficients of Equations Contain Hyperbolic Tangent 100
4.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 101
4.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 102
4.5. Equations Containing Logarithmic Functions 102
4.5.1. Coefficients of Equations Contain Logarithmic Functions 102
4.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 103
4.6. Equations Containing Trigonometric Functions 104
4.6.1. Coefficients of Equations Contain Sine 104
4.6.2. Coefficients of Equations Contain Cosine 104
4.6.3. Coefficients of Equations Contain Tangent 105
4.6.4. Coefficients of Equations Contain Cotangent 106
4.6.5. Coefficients of Equations Contain Different Trigonometric Functions 106
4.7. Equations Containing Inverse Trigonometric Functions 107
4.7.1. Coefficients of Equations Contain Arcsine 107
4.7.2. Coefficients of Equations Contain Arccosine 108
4.7.3. Coefficients of Equations Contain Arctangent 108
4.7.4. Coefficients of Equations Contain Arccotangent 109
4.8. Equations Containing Arbitrary Functions 109
4.8.1. Coefficients of Equations Contain Arbitrary Functions of x 109
4.8.2. Equations Contain Arbitrary Functions of x and Arbitrary Functions of y 112
4.8.3. Equations Contain Arbitrary Functions of Complicated Arguments 113
4.8.4. Equations Contain Arbitrary Functions of Two Variables 114
5. Linear Equations of the Form $f(x,y)\frac{\partial w}{\partial x} + g(x,y)\frac{\partial w}{\partial y} = h_1(x,y)w + h_0(x,y)$ 115
5.1. Preliminary Remarks 115
5.1.1. Solution Methods 115
5.1.2. Examples 116
5.2. Equations Containing Power-Law Functions 117
5.2.1. Coefficients of Equations Are Linear in x and y 117
5.2.2. Coefficients of Equations Are Quadratic in x and y 118
5.2.3. Coefficients of Equations Contain Square Roots 119
5.2.4. Coefficients of Equations Contain Arbitrary Powers of x and y 120
5.3. Equations Containing Exponential Functions 122
5.3.1. Coefficients of Equations Contain Exponential Functions 122
5.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 123

5.4. Equations Containing Hyperbolic Functions 125
5.4.1. Coefficients of Equations Contain Hyperbolic Sine 125
5.4.2. Coefficients of Equations Contain Hyperbolic Cosine 125
5.4.3. Coefficients of Equations Contain Hyperbolic Tangent 126
5.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 126
5.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 127
5.5. Equations Containing Logarithmic Functions 127
5.5.1. Coefficients of Equations Contain Logarithmic Functions 127
5.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 128
5.6. Equations Containing Trigonometric Functions 129
5.6.1. Coefficients of Equations Contain Sine 129
5.6.2. Coefficients of Equations Contain Cosine 130
5.6.3. Coefficients of Equations Contain Tangent 130
5.6.4. Coefficients of Equations Contain Cotangent 131
5.6.5. Coefficients of Equations Contain Different Trigonometric Functions 132
5.7. Equations Containing Inverse Trigonometric Functions 133
5.7.1. Coefficients of Equations Contain Arcsine 133
5.7.2. Coefficients of Equations Contain Arccosine 133
5.7.3. Coefficients of Equations Contain Arctangent 134
5.7.4. Coefficients of Equations Contain Arccotangent 134
5.8. Equations Containing Arbitrary Functions 135
5.8.1. Coefficients of Equations Contain Arbitrary Functions of x 135
5.8.2. Equations Contain Arbitrary Functions of x and Arbitrary Functions of y 137
5.8.3. Equations Contain Arbitrary Functions of Two Variables 138

Part II. Linear Equations With Three or More Independent Variables 139

6. Linear Equations of the Form $f(x,y,z)\frac{\partial w}{\partial x} + g(x,y,z)\frac{\partial w}{\partial y} + h(x,y,z)\frac{\partial w}{\partial z} = 0$ 141
6.1. Preliminary Remarks 141
6.1.1. Solution Methods 141
6.1.2. Cauchy Problem (Initial Value Problem) 142
6.1.3. Examples 142
6.2. Equations Containing Power-Law Functions 143
6.2.1. Coefficients of Equations Are Linear in x, y, and z 143
6.2.2. Coefficients of Equations Are Quadratic in x, y, and z 147
6.2.3. Coefficients of Equations Contain Other Powers of x, y, and z 150
6.2.4. Coefficients of Equations Contain Arbitrary Powers of x, y, and z 151
6.3. Equations Containing Exponential Functions 154
6.3.1. Coefficients of Equations Contain Exponential Functions 154
6.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 155
6.4. Equations Containing Hyperbolic Functions 157
6.4.1. Coefficients of Equations Contain Hyperbolic Sine 157
6.4.2. Coefficients of Equations Contain Hyperbolic Cosine 158
6.4.3. Coefficients of Equations Contain Hyperbolic Tangent 159
6.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 159
6.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 160
6.5. Equations Containing Logarithmic Functions 161
6.5.1. Coefficients of Equations Contain Logarithmic Functions 161
6.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 161

6.6. Equations Containing Trigonometric Functions 162
6.6.1. Coefficients of Equations Contain Sine 162
6.6.2. Coefficients of Equations Contain Cosine 162
6.6.3. Coefficients of Equations Contain Tangent 163
6.6.4. Coefficients of Equations Contain Cotangent 163
6.6.5. Coefficients of Equations Contain Different Trigonometric Functions 164
6.7. Equations Containing Inverse Trigonometric Functions 164
6.7.1. Coefficients of Equations Contain Arcsine 164
6.7.2. Coefficients of Equations Contain Arccosine 165
6.7.3. Coefficients of Equations Contain Arctangent 166
6.7.4. Coefficients of Equations Contain Arccotangent 166
6.8. Equations Containing Arbitrary Functions 167
6.8.1. Coefficients of Equations Contain Arbitrary Functions of x 167
6.8.2. Coefficients of Equations Contain Arbitrary Functions of Different Variables . . 169
6.8.3. Coefficients of Equations Contain Arbitrary Functions of Two Variables 170
7. Linear Equations of the Form $f_1\frac{\partial w}{\partial x} + f_2\frac{\partial w}{\partial y} + f_3\frac{\partial w}{\partial z} = f_4$, $f_n = f_n(x, y, z)$ 173
7.1. Preliminary Remarks 173
7.1.1. Solution Methods 173
7.1.2. Examples 174
7.2. Equations Containing Power-Law Functions 175
7.2.1. Coefficients of Equations Are Linear in x, y, and z 175
7.2.2. Coefficients of Equations Are Quadratic in x, y, and z 176
7.2.3. Coefficients of Equations Contain Other Powers in x, y, and z 177
7.2.4. Coefficients of Equations Contain Arbitrary Powers of x, y, and z 178
7.3. Equations Containing Exponential Functions 180
7.3.1. Coefficients of Equations Contain Exponential Functions 180
7.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 181
7.4. Equations Containing Hyperbolic Functions 182
7.4.1. Coefficients of Equations Contain Hyperbolic Sine 182
7.4.2. Coefficients of Equations Contain Hyperbolic Cosine 183
7.4.3. Coefficients of Equations Contain Hyperbolic Tangent 184
7.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 185
7.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 186
7.5. Equations Containing Logarithmic Functions 186
7.5.1. Coefficients of Equations Contain Logarithmic Functions 186
7.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 187
7.6. Equations Containing Trigonometric Functions 187
7.6.1. Coefficients of Equations Contain Sine 187
7.6.2. Coefficients of Equations Contain Cosine 188
7.6.3. Coefficients of Equations Contain Tangent 189
7.6.4. Coefficients of Equations Contain Cotangent 190
7.6.5. Coefficients of Equations Contain Different Trigonometric Functions 190
7.7. Equations Containing Inverse Trigonometric Functions 191
7.7.1. Coefficients of Equations Contain Arcsine 191
7.7.2. Coefficients of Equations Contain Arccosine 192
7.7.3. Coefficients of Equations Contain Arctangent 192
7.7.4. Coefficients of Equations Contain Arccotangent 193

7.8. Equations Containing Arbitrary Functions 193
7.8.1. Coefficients of Equations Contain Arbitrary Functions of x 193
7.8.2. Coefficients of Equations Contain Arbitrary Functions of Different Variables . . 195
7.8.3. Coefficients of Equations Contain Arbitrary Functions of Two Variables 196

8. Linear Equations of the Form $f_1\frac{\partial w}{\partial x} + f_2\frac{\partial w}{\partial y} + f_3\frac{\partial w}{\partial z} = f_4 w,\ \ f_n = f_n(x, y, z)$ 199

8.1. Preliminary Remarks 199
8.1.1. Solution Methods 199
8.1.2. Examples 200
8.2. Equations Containing Power-Law Functions 201
8.2.1. Coefficients of Equations Are Linear in x, y, and z 201
8.2.2. Coefficients of Equations Are Quadratic in x, y, and z 202
8.2.3. Coefficients of Equations Contain Other Powers of x, y, and z 203
8.2.4. Coefficients of Equations Contain Arbitrary Powers of x, y, and z 204
8.3. Equations Containing Exponential Functions 206
8.3.1. Coefficients of Equations Contain Exponential Functions 206
8.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 207
8.4. Equations Containing Hyperbolic Functions 208
8.4.1. Coefficients of Equations Contain Hyperbolic Sine 208
8.4.2. Coefficients of Equations Contain Hyperbolic Cosine 209
8.4.3. Coefficients of Equations Contain Hyperbolic Tangent 210
8.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 211
8.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 211
8.5. Equations Containing Logarithmic Functions 212
8.5.1. Coefficients of Equations Contain Logarithmic Functions 212
8.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 213
8.6. Equations Containing Trigonometric Functions 213
8.6.1. Coefficients of Equations Contain Sine 213
8.6.2. Coefficients of Equations Contain Cosine 214
8.6.3. Coefficients of Equations Contain Tangent 215
8.6.4. Coefficients of Equations Contain Cotangent 215
8.6.5. Coefficients of Equations Contain Different Trigonometric Functions 216
8.7. Equations Containing Inverse Trigonometric Functions 217
8.7.1. Coefficients of Equations Contain Arcsine 217
8.7.2. Coefficients of Equations Contain Arccosine 217
8.7.3. Coefficients of Equations Contain Arctangent 218
8.7.4. Coefficients of Equations Contain Arccotangent 218
8.8. Equations Containing Arbitrary Functions 219
8.8.1. Coefficients of Equations Contain Arbitrary Functions of x 219
8.8.2. Coefficients of Equations Contain Arbitrary Functions of Different Variables . . 221
8.8.3. Coefficients of Equations Contain Arbitrary Functions of Two Variables 222

9. Linear Equations of the Form $f_1\frac{\partial w}{\partial x} + f_2\frac{\partial w}{\partial y} + f_3\frac{\partial w}{\partial z} = f_4 w + f_5,\ \ f_n = f_n(x, y, z)$ 225

9.1. Preliminary Remarks 225
9.1.1. Solution Methods 225
9.1.2. Examples 226
9.2. Equations Containing Power-Law Functions 226
9.2.1. Coefficients of Equations Are Linear in x, y, and z 226
9.2.2. Coefficients of Equations Are Quadratic in x, y, and z 228
9.2.3. Coefficients of Equations Contain Other Powers of x, y, and z 228
9.2.4. Coefficients of Equations Contain Arbitrary Powers of x, y, and z 229

9.3. Equations Containing Exponential Functions 231
9.3.1. Coefficients of Equations Contain Exponential Functions 231
9.3.2. Coefficients of Equations Contain Exponential and Power-Law Functions 232
9.4. Equations Containing Hyperbolic Functions 233
9.4.1. Coefficients of Equations Contain Hyperbolic Sine 233
9.4.2. Coefficients of Equations Contain Hyperbolic Cosine 234
9.4.3. Coefficients of Equations Contain Hyperbolic Tangent 235
9.4.4. Coefficients of Equations Contain Hyperbolic Cotangent 235
9.4.5. Coefficients of Equations Contain Different Hyperbolic Functions 236
9.5. Equations Containing Logarithmic Functions 237
9.5.1. Coefficients of Equations Contain Logarithmic Functions 237
9.5.2. Coefficients of Equations Contain Logarithmic and Power-Law Functions 237
9.6. Equations Containing Trigonometric Functions 238
9.6.1. Coefficients of Equations Contain Sine 238
9.6.2. Coefficients of Equations Contain Cosine 239
9.6.3. Coefficients of Equations Contain Tangent 240
9.6.4. Coefficients of Equations Contain Cotangent 240
9.6.5. Coefficients of Equations Contain Different Trigonometric Functions 241
9.7. Equations Containing Inverse Trigonometric Functions 242
9.7.1. Coefficients of Equations Contain Arcsine 242
9.7.2. Coefficients of Equations Contain Arccosine 242
9.7.3. Coefficients of Equations Contain Arctangent 243
9.7.4. Coefficients of Equations Contain Arccotangent 243
9.8. Equations Containing Arbitrary Functions 244
9.8.1. Coefficients of Equations Contain Arbitrary Functions of x 244
9.8.2. Coefficients of Equations Contain Arbitrary Functions of Different Variables . . 245
9.8.3. Coefficients of Equations Contain Arbitrary Functions of Two Variables 246
10. Linear Equations With Four or More Independent Variables 249
10.1. Preliminary Remarks 249
10.1.1. Linear Homogeneous Equations 249
10.1.2. Linear Nonhomogeneous Equations 250
10.2. Specific Equations 251
10.2.1. Equations Containing Power-Law Functions 251
10.2.2. Other Equations Containing Arbitrary Parameters 254
10.2.3. Equations Containing Arbitrary Functions 256
Part III. Nonlinear Equations 259
11. Quasilinear Equations of the Form $f(x,y)\frac{\partial w}{\partial x} + g(x,y)\frac{\partial w}{\partial y} = h(x,y,w)$ 261
11.1. Preliminary Remarks 261
11.1.1. Solution Methods 261
11.1.2. Examples 262
11.2. Equations Containing Arbitrary Parameters 263
11.2.1. Coefficients of Equations Contain Power-Law Functions 263
11.2.2. Coefficients of Equations Contain Exponential Functions 264
11.2.3. Coefficients of Equations Contain Hyperbolic Functions 266
11.2.4. Coefficients of Equations Contain Logarithmic Functions 267
11.2.5. Coefficients of Equations Contain Trigonometric Functions 268
11.3. Equations Containing Arbitrary Functions 268
11.3.1. Equations Contain Arbitrary Functions of One Variable 268
11.3.2. Equations Contain Arbitrary Functions of Two Variables 271

12. Quasilinear Equations of the Form $f(x,y,w)\frac{\partial w}{\partial x}+g(x,y,w)\frac{\partial w}{\partial y}=h(x,y,w)$ . . . 273
12.1. Preliminary Remarks . . . 273
12.1.1. Solution Methods . . . 273
12.1.2. Cauchy Problem. Existence and Uniqueness Theorem . . . 274
12.1.3. Equation $\frac{\partial w}{\partial x}+f(w)\frac{\partial w}{\partial y}=0$. Qualitative Features and Discontinuous Solutions 276
12.1.4. Generalized Solutions of Quasilinear Equations . . . 286
12.2. Equations Containing Power-Law Functions . . . 290
12.2.1. Coefficients of Equations Are Linear in w . . . 290
12.2.2. Coefficients of Equations Are Quadratic in w . . . 293
12.2.3. Coefficients of Equations Contain Other Powers of w . . . 295
12.3. Other Equations Containing Arbitrary Parameters . . . 297
12.3.1. Coefficients of Equations Contain Exponential Functions . . . 297
12.3.2. Coefficients of Equations Contain Hyperbolic Functions . . . 299
12.3.3. Coefficients of Equations Contain Logarithmic Functions . . . 302
12.3.4. Coefficients of Equations Contain Trigonometric Functions . . . 304
12.4. Equations Containing Arbitrary Functions . . . 306
12.4.1. Equations Contain Arbitrary Functions of Independent Variables . . . 306
12.4.2. Equations Contain Arbitrary Functions of the Unknown Variable . . . 310
12.4.3. Equations Contain Arbitrary Functions of Two Variables . . . 314

13. Equations With Two Independent Variables Quadratic in Derivatives . . . 317
13.1. Preliminary Remarks . . . 317
13.2. Equations Containing Arbitrary Parameters . . . 317
13.2.1. Equations of the Form $\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}=f(x,y,w)$ . . . 317
13.2.2. Equations of the Form $f(x,y,w)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+g(x,y,w)\frac{\partial w}{\partial x}=h(x,y,w)$ . . . 319
13.2.3. Equations of the Form $f(x,y,w)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+g(x,y,w)\frac{\partial w}{\partial x}+h(x,y,w)\frac{\partial w}{\partial y}=s(x,y,w)$ . . . 321
13.2.4. Equations of the Form $\frac{\partial w}{\partial x}+f(x,y,w)\big(\frac{\partial w}{\partial y}\big)^2=g(x,y,w)$ . . . 324
13.2.5. Equations of the Form $\frac{\partial w}{\partial x}+f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^2+g(x,y,w)\frac{\partial w}{\partial y}=h(x,y,w)$ . . . 331
13.2.6. Equations of the Form $f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^2+g(x,y,w)\big(\frac{\partial w}{\partial y}\big)^2=h(x,y,w)$ . . . 335
13.2.7. Equations of the Form $f(x,y)\big(\frac{\partial w}{\partial x}\big)^2+g(x,y)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}=h(x,y,w)$ . . . 340
13.2.8. Other Equations . . . 343
13.3. Equations Containing Arbitrary Functions . . . 347
13.3.1. Equations of the Form $\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}=f(x,y,w)$ . . . 347
13.3.2. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+g(x,y)\frac{\partial w}{\partial x}=h(x,y,w)$ . . . 349
13.3.3. Equations of the Form $f(x,y)\frac{\partial w}{\partial x}+g(x,y,w)\big(\frac{\partial w}{\partial y}\big)^2=h(x,y,w)$ . . . 351
13.3.4. Equations of the Form $\frac{\partial w}{\partial x}+f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^2+g(x,y,w)\frac{\partial w}{\partial y}=h(x,y,w)$ . . . 356
13.3.5. Equations of the Form $f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^2+g(x,y,w)\big(\frac{\partial w}{\partial y}\big)^2=h(x,y,w)$ . . . 359
13.3.6. Equations of the Form $\big(\frac{\partial w}{\partial x}\big)^2+f(x,y,w)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}=g(x,y,w)$ . . . 361
13.3.7. Other Equations . . . 364

14. Nonlinear Equations With Two Independent Variables of General Form . . . 367
14.1. Preliminary Remarks . . . 367
14.1.1. Solution Methods . . . 367
14.1.2. Cauchy Problem. Existence and Uniqueness Theorem . . . 371
14.1.3. Generalized Viscosity Solutions and Their Applications . . . 374

14.2. Equations Containing Cubic Nonlinearities With Respect to Derivatives 378
14.2.1. Equations of the Form $\frac{\partial w}{\partial x}\big(\frac{\partial w}{\partial y}\big)^2 = f(x,y,w)$ 378
14.2.2. Equations of the Form $f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^3 + g(x,y,w)\frac{\partial w}{\partial y} = h(x,y,w)$ 379
14.2.3. Equations of the Form $f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^3 + g(x,y,w)\big(\frac{\partial w}{\partial y}\big)^2 = h(x,y,w)$ 381
14.2.4. Equations of the Form $f(x,y,w)\big(\frac{\partial w}{\partial x}\big)^3 + g(x,y,w)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = h(x,y,w)$ 382
14.2.5. Other Equations 383
14.3. Nonlinear Equations Containing Arbitrary Parameters 385
14.3.1. Equations Contain the Fourth Powers of Derivatives 385
14.3.2. Equations Contain Derivatives in Radicands 387
14.3.3. Equations Contain Arbitrary Powers of Derivatives 387
14.3.4. More Complicated Equations 390
14.4. Equations Containing Arbitrary Functions of Independent Variables 392
14.4.1. Equations Contain One Arbitrary Power of Derivative 392
14.4.2. Equations Contain Two or Three Arbitrary Powers of Derivatives 395
14.5. Equations Containing Arbitrary Functions of Derivatives 397
14.5.1. Equations Contain Arbitrary Functions of One Variable 397
14.5.2. Equations Contain Arbitrary Functions of Two Variables 400
14.5.3. Equations Contain Arbitrary Functions of Three Variables 404
14.5.4. Equations Contain Arbitrary Functions of Four Variables 406

15. Nonlinear Equations With Three or More Independent Variables 407

15.1. Preliminary Remarks 407
15.1.1. Quasilinear Equations 407
15.1.2. Nonlinear Equations 409
15.1.3. Generalized Viscosity Solutions 416
15.2. Quasilinear Equations 418
15.2.1. Equations With Three Variables 418
15.2.2. Equations With Arbitrary Number of Variables 422
15.3. Nonlinear Equations With Three Variables Quadratic in Derivatives 424
15.3.1. Equations Contain Squares of One or Two Derivatives 424
15.3.2. Equations Contain Squares of Three Derivatives 429
15.3.3. Equations Contain Products of Derivatives With Respect to Different Variables 430
15.3.4. Equations Contain Squares and Products of Derivatives 432
15.4. Other Nonlinear Equations With Three Variables Containing Parameters 432
15.4.1. Equations Cubic in Derivatives 432
15.4.2. Equations Contain Roots and Moduli of Derivatives 433
15.4.3. Equations Contain Arbitrary Powers of Derivatives 434
15.5. Nonlinear Equations With Three Variables Containing Arbitrary Functions 437
15.5.1. Equations Quadratic in Derivatives 437
15.5.2. Equations With Power Nonlinearity in Derivatives 443
15.5.3. Equations With Arbitrary Dependence on Derivatives 445
15.5.4. Nonlinear Equations of General Form 446
15.6. Nonlinear Equations With Four Independent Variables 450
15.6.1. Equations Quadratic in Derivatives 450
15.6.2. Equations Contain Power-Law Functions of Derivatives 452
15.7. Nonlinear Equations With Arbitrary Number of Variables Containing Arbitrary Parameters 454
15.7.1. Equations Quadratic in Derivatives 454
15.7.2. Equations With Power-Law Nonlinearity in Derivatives 456

15.8. Nonlinear Equations With Arbitrary Number of Variables Containing Arbitrary Functions 457
15.8.1. Equations Quadratic in Derivatives 457
15.8.2. Equations With Power-Law Nonlinearity in Derivatives 462
15.8.3. Equations Contain Arbitrary Functions of Two Variables 463
15.8.4. Nonlinear Equations of General Form 464

Supplement. Solution of Differential Equations Through the CONVODE Software 469

S.1. Introduction 469
S.1.1. Preliminary Remarks 469
S.1.2. Reduce Notation Used in CONVODE 469
S.1.3. How CONVODE Solves Equations 470
S.2. Examples of Solving Ordinary Differential Equations 471
S.2.1. Riccati Equation (Example 1) 471
S.2.2. Riccati Equation (Example 2) 475
S.2.3. A Nonlinear Equation Quadratic in the Derivative 479
S.3. Examples of Solving Partial Differential Equations 481
S.3.1. A First Order Linear Equation (Example 1) 481
S.3.2. A First Order Linear Equation (Example 2) 483
S.3.3. A Second Order Nonlinear Equation 486
S.4. How to Use CONVODE 490
S.4.1. Arguments of the CONVODE procedure 490
S.4.2. Global variables 491
S.4.3. CONVODE via e-mail 492

References 493

Index 497