FOREWORD

Nonlinear partial differential equations are encountered in various fields of mathematics, physics, chemistry, and biology, and numerous applications.

Exact (closed-form) solutions of differential equations play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear equations graphically demonstrate and allow unraveling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiplicity or absence steady states under various conditions, existence of peaking regimes and many others. Furthermore, simple solutions are often used in teaching many courses as specific examples illustrating basic tenets of a theory that admit mathematical formulation.

Even those special exact solutions that do not have a clear physical meaning can be used as "test problems" to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving differential equations. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions.

This book contains more than 1600 nonlinear mathematical physics equations and nonlinear partial differential equations and their solutions. A large number of new exact solutions to nonlinear equations are described. Equations of parabolic, hyperbolic, elliptic, mixed, and general types are discussed. Second-, third-, fourth-, and higher-order nonlinear equations are considered. The book presents exact solutions to equations of heat and mass transfer, wave theory, nonlinear mechanics, hydrodynamics, gas dynamics, plasticity theory, nonlinear acoustics, combustion theory, nonlinear optics, theoretical physics, differential geometry, control theory, chemical engineering sciences, biology, and other fields.

Special attention is paid to general-form equations that depend on arbitrary functions; exact solutions of such equations are of principal value for testing numerical and approximate methods. Almost all other equations contain one or more arbitrary parameters (in fact, this book deals with whole families of partial differential equations), which can be fixed by the reader at will. *In total, the handbook contains significantly more nonlinear PDE's and exact solutions than any other book currently available*.

The supplement of the book presents exact analytical methods for solving nonlinear mathematical physics equations. When selecting the material, the authors have given a pronounced preference to practical aspects of the matter; that is, to methods that allow effectively "constructing" exact solutions. Apart from the classical methods, the book also describes wide-range methods that have been greatly developed over the last decade (the nonclassical and direct methods for symmetry reductions, the differential constraints method, the method of generalized separation of variables, and others). For the reader's better understanding of the methods, numerous examples of solving specific differential equations and systems of differential equations are given throughout the book.

For the convenience of a wide audience with different mathematical backgrounds, the authors tried to do their best, wherever possible, to avoid special terminology. Therefore, some of the methods are outlined in a schematic and somewhat simplified manner, with necessary references made to books where these methods are considered in more detail. Many sections were written so that they could be read independently from each other. This allows the reader to quickly get to the heart of the matter.

The handbook consists of chapters, sections, and subsections. Equations and formulas are numbered separately in each subsection. The equations within subsections are arranged in increasing order of complexity. The extensive table of contents provides rapid access to the desired equations.

Separate parts of the book may be used by lecturers of universities and colleges for practical courses and lectures on nonlinear mathematical physics equations for graduate and postgraduate students. Furthermore, the books may be used as a database of test problems for numerical and approximate methods for solving nonlinear partial differential equations.

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The authors hope that this book will be helpful for a wide range of scientists, university teachers, engineers, and students engaged in the fields of mathematics, physics, mechanics, control, chemistry, and engineering sciences.

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