



## INDEX

### A

Abel equation (ODE), 5, 7, 8, 31, 37  
additive separable solution, xviii, 698  
adiabatic gas flow, 776, 778, 781, 785  
Airy equation (ODE), 518  
auto-Bäcklund transformation, 126, 691  
autonomous equation (ODE), 1–3, 5–8, 11

### B

Bäcklund transformation, 11, 267, 519, 690–693  
  auto-, 126, 691  
BBM equation, 583  
Bellman type equations, 472–477  
Bernoulli equation (ODE), 77, 78, 84, 87, 90  
bilinear functional equations, 705, 706  
Blasius problem, 542  
  generalized, 548  
Blasius solution, 125  
Born–Infeld equation, 445  
boundary layer,  
  diffusion, 118, 124, 168  
  hydrodynamic, 540–547, 549, 551, 553  
  thermal, 124, 125  
Boussinesq equation, 142, 595, 597  
  canonical form, 595  
  unnormalized, 597  
Burgers equation, 9, 78, 79  
  cylindrical, 14  
  generalized, 10  
  modified, 13  
  unnormalized, 11, 109  
  vector, 179  
Burgers–Korteweg–de Vries equation, 532  
  generalized, 635

### C

Calogero equation, 434  
canonical form  
  Boussinesq equation, 595  
  elliptic equations, 684  
  hyperbolic equations, 684  
  Korteweg–de Vries equation, 515  
  parabolic equations, 683  
Cauchy problem, 10, 520, 446, 765, 773  
Cauchy–Riemann equations, 739  
characteristic direction, 774  
characteristic equation, 683  
characteristic lines, 773, 774  
characteristic velocity, 774  
Clarkson–Kruskal direct method, 728, 731  
classical method for symmetry reductions, 735  
classification of second-order semilinear equations,  
  683

cnoidal waves, 515  
commutator of operators, 765  
condition of conservation of energy, 31  
conditions  
  evolutionary, 780  
  Rankine–Hugoniot jump, 779  
conservation laws, 106, 108, 693, 769, 772  
  mass, 772, 777  
  momentum, 772, 777  
  systems, 772  
contact transformations, 688  
continuity equation, 772  
continuous point Lie group, 736  
coordinates of first prolongation, 737–740, 742,  
  745  
coordinates of second prolongation, 737–740, 742,  
  745  
Crocco transformation, 543, 546  
cylindrical Burgers equation, 14  
cylindrical Korteweg–de Vries equation, 521

### D

differential constraints  
  first-order, 749  
  method, 747–749  
  second-order, 754  
  third- and higher-order, 756  
differential substitutions, 691  
differentiation method, 718  
double sine-Gordon equation, 229  
double sinh-Gordon equation, 225

### E

eigenvalue of matrix, 774  
eigenvector of matrix  
  left, 774–777  
  right, 774, 777, 780, 787  
elliptic equations  
  canonical form, 684  
  with  $n$  independent variables, 428–431  
  with three space variables, 405–428  
  with two space variables, 347–404  
elliptic Weierstrass function, 2, 362, 533, 596, 598  
Emden–Fowler equation (ODE), 48, 198, 199,  
  210, 211  
  generalized (ODE), 111, 114, 202, 219, 245  
equation  
  Abel (ODE), 5, 7, 8, 31, 37  
  Airy (ODE), 518  
  anisotropic diffusion, stationary, 395  
  anisotropic heat, stationary, 395  
  autonomous (ODE), 1–3, 5–8, 11  
  BBM, 583  
  Bernoulli (ODE), 77, 78, 84, 87, 90

- Born–Infeld, 445  
 Boussinesq, 142, 595, 597  
 Boussinesq, canonical form, 595  
 Boussinesq, unnormalized, 597  
 Burgers, 9, 78, 79  
 Burgers, cylindrical, 14  
 Burgers, generalized, 10  
 Burgers–Korteweg–de Vries, 532  
 Burgers–Korteweg–de Vries, generalized, 635  
 Burgers, modified, 13  
 Burgers, unnormalized, 11, 109  
 Burgers, vector, 179  
 Calogero, 434  
 characteristic, 683  
 continuity, 772  
 cylindrical Burgers, 14  
 cylindrical Korteweg–de Vries, 521  
 diffusion boundary layer, nonlinear, 118, 124, 168, 169  
 double sine–Gordon, 229  
 double sinh–Gordon, 225  
 Emden–Fowler (ODE), 48, 198, 199, 210, 211  
 Emden–Fowler, generalized (ODE), 111, 114, 202, 219, 245  
 first Painlevé (ODE), 518, 596, 759  
 Fisher, 1  
 Fitzhugh–Nagumo, 4, 744  
 Gel’fand–Levitan–Marchenko, integral, 519, 520, 523, 597, 606  
 generalized Burgers–Korteweg–de Vries, 635  
 generalized Kadomtsev–Petviashvili, 629  
 generalized Khokhlov–Zabolotskaya, 437  
 generalized Korteweg–de Vries, 524  
 generalized Landau–Ginzburg, 135  
 generalized Liouville, 679  
 Goursat, 272  
 Grad–Shafranov, 379  
 Harry Dym, 528, 732  
 Helmholtz, 150, 165, 182, 184, 279  
 homogeneous (ODE), 23, 24, 35, 49, 205  
 homogeneous Monge–Ampère, 451  
 hyperbolic, 684  
 Kadomtsev–Petviashvili, 605, 762  
 Kadomtsev–Petviashvili, canonical form, 605  
 Kadomtsev–Petviashvili, generalized, 629  
 Kadomtsev–Petviashvili, unnormalized, 607  
 Kawahara, 632  
 Khokhlov–Zabolotskaya, 435  
 Khokhlov–Zabolotskaya, generalized, 437  
 Khokhlov–Zabolotskaya, stationary, 353  
 Khokhlov–Zabolotskaya, three-dimensional, 438  
 Khokhlov–Zabolotskaya, two-dimensional, 435  
 Klein–Gordon, nonlinear, 234  
 Kolmogorov–Petrovskii–Piskunov, 71  
 Korteweg–de Vries, 515, 767  
 Korteweg–de Vries, canonical form, 515  
 Korteweg–de Vries, cylindrical, 521  
 Korteweg–de Vries, generalized, 524  
 Korteweg–de Vries, modified, 522  
 Korteweg–de Vries, spherical, 522  
 Korteweg–de Vries, unnormalized, 523  
 Kuramoto–Sivashinsky, 593  
 Laplace, 162, 173, 174, 182, 278  
 linear wave, 684  
 Liouville, 213, 267  
 Liouville, generalized, 679  
 minimal surfaces, 447  
 modified Burgers, 13  
 modified Korteweg–de Vries, 522  
 Monge–Ampère, 451  
 nonlinear diffusion boundary layer, 124, 168, 169  
 nonlinear Klein–Gordon, 234, 719, 721  
 Painlevé, first (ODE), 518, 596, 759  
 Painlevé, second (ODE), 518, 523, 596, 759  
 Poisson, 150, 155, 163, 174, 184  
 Riccati (ODE), 75, 77, 78, 81, 85  
 Schrödinger, of general form, 134, 135, 137, 138  
 Schrödinger, three-dimensional, of general form, 189  
 Schrödinger, two-dimensional, of general form, 188  
 Schrödinger, with cubic nonlinearity, 125, 127, 128, 129, 131, 189  
 Schrödinger, with power-law nonlinearity, 127, 129  
 separable (ODE), 35, 47, 208, 214, 366  
 sine–Gordon, 227, 229, 269, 766  
 sine–Gordon, double, 229  
 sinh–Gordon, 225, 268, 767  
 sinh–Gordon, double, 225  
 state, ideal polytropic gas, 772  
 stationary anisotropic diffusion, 395  
 stationary anisotropic heat, 395  
 stationary heat, with nonlinear source, 382  
 stationary Khokhlov–Zabolotskaya, 353  
 steady transonic gas flow, 361, 363, 431  
 thermal boundary layer, nonlinear, 124  
 Tzitzéica, 268  
 unnormalized Boussinesq, 597  
 unnormalized Burgers, 11  
 unsteady transonic gas flow, 409, 440  
 vector Burgers, 179  
 Yermakov (ODE), 97, 251
- equations  
 admitting variational formulation, 772  
 Bellman type, 472–477  
 bilinear functional, 705, 706  
 boundary layer, 118, 124, 125, 540, 553  
 boundary layer, for Newtonian fluid, 553  
 boundary layer, for non-Newtonian fluids, 547, 566  
 elliptic, canonical form, 684  
 elliptic, with  $n$  independent variables, 428–431  
 elliptic, with three space variables, 405–428  
 elliptic, with two space variables, 347–404  
 Euler, 570, 573, 574  
 Euler–Lagrange, 770  
 fifth-order, 631–633  
 fourth-order, 589–629  
 functional, 705, 723–728

- functional, bilinear, 705, 706  
 functional, three-argument, 722  
 functional-differential, 701  
 heat and mass transfer, 159, 174, 391, 420, 423  
 heat and mass transfer in anisotropic media,  
   161, 163, 164, 166, 167  
 heat transfer in quiescent media, 29, 53  
 heat transfer in quiescent media with chemical  
   reaction, 107, 159, 160  
 heat and mass transfer with complicating fac-  
   tors, 423  
 higher orders, 631–681  
 hydrodynamic boundary layer, 542  
 hydrodynamic, canonical form, 684  
 hyperbolic, with one space variable, 191–273  
 hyperbolic, with three space variables, 317–346  
 hyperbolic, with two space variables, 275–316  
 involving arbitrary functions, 71, 125, 161, 236,  
   273, 384  
 mass transfer in moving media with chemical  
   reactions, 169  
 mass transfer in quiescent media with chemical  
   reactions, 159, 169  
 motion of ideal fluid, 570  
 Navier–Stokes, nonstationary, 616  
 Navier–Stokes, stationary, 607  
 nonlinear diffusion boundary layer, 124, 168,  
   169  
 nonlinear functional, 723  
 nonlinear Schrödinger, 125–130, 186–190  
 nonlinear telegraph, 299  
 nonstationary, 155, 157  
 nonstationary hydrodynamic, 574, 616, 627  
 parabolic, canonical form, 683  
 parabolic, with one space variable, 1–140  
 parabolic, with three space variables, 169–179  
 parabolic, with two space variables, 141–168  
 reducible to Korteweg–de Vries equation, 526  
 second-order, 1–588  
 second-order, evolution, 749  
 second-order, general form, 479–514  
 second-order, semilinear, 683  
 stationary hydrodynamic, 570, 607  
 steady boundary layer, for Newtonian fluid, 540  
 steady boundary layer, for non-Newtonian flu-  
   ids, 547  
 telegraph, linear, 230, 265, 300, 301, 303  
 telegraph, nonlinear, 299–305  
 third-order, 515–588  
 unsteady boundary layer, for Newtonian fluid,  
   553  
 unsteady boundary layer, for non-Newtonian  
   fluids, 564  
 elliptic equations  
   canonical form, 684  
   with  $n$  independent variables, 428–431  
   with three space variables, 405–428  
   with two space variables, 347–404  
 error function, 10  
 Euler equations, 570, 573, 574  
 Euler–Lagrange equation, 770  
 Euler transformation, 123, 447, 689  
 evolutionary conditions, 780  
 evolutionary shocks, 780  
 exponential self-similar solutions, 696
- ## F
- Falkner–Skan problem, 544  
   generalized, 550  
 fifth-order equations, 631–633  
 first-order differential constraints, 749  
 first Painlevé equation (ODE), 518, 596, 759  
 first prolongation, coordinates of, 737, 738  
 Fisher equation, 1  
 Fitzhugh–Nagumo equation, 4, 744  
 fixed singularities of solutions, 758  
 fourth-order equations, 589–629  
 function  
   error, 10  
   Weierstrass elliptic, 2, 533  
 functional-differential equations, 701, 705  
 functional equation, 723–728  
   bilinear, 705, 706  
 functional separable solutions, xviii, 713
- ## G
- gas flow, adiabatic, 776, 778, 781, 785  
 gas, ideal polytropic, equation of state, 772  
 Gel'fand–Levitan–Marchenko integral equation,  
   519, 520, 523, 597, 606  
 generalized Blasius problem, 548  
 generalized Burgers–Korteweg–de Vries equation,  
   635  
 generalized Kadomtsev–Petviashvili equation, 629  
 generalized Khokhlov–Zabolotskaya equation, 437  
 generalized Korteweg–de Vries equation, 524  
 generalized Landau–Ginzburg equation, 135  
 generalized Liouville equation, 679  
 generalized Schlichting problem, 548  
 generalized separable solutions, xviii, 701, 709  
 generalized similarity reductions, 728, 757  
 generalized traveling-wave solutions, xviii, 713  
 Goursat equation, 272  
 Grad–Shafranov equation, 379  
 group analysis methods, 735–747  
   classical, 735  
   nonclassical, 744  
 group invariant, 737, 741
- ## H
- Harry Dym equation, 528, 732  
 heat and mass transfer equations,  
   in anisotropic media, 161, 163, 164, 166, 167  
   in moving media with chemical reactions, 169  
   in quiescent media, 29, 53  
   in quiescent media with chemical reactions,  
     107, 159, 169  
   with complicating factors, 423  
 Helmholtz equation, 150, 165, 182, 184, 279  
 hodograph transformation, 26, 122, 254, 356, 686  
 homogeneous equation (ODE), 23, 24, 35, 49, 205

homogeneous Monge–Ampère equation, 451  
 Hopf–Cole transformation, 10, 692, 762  
 hydrodynamic boundary layer equations, 540, 544, 546, 547, 549  
 hyperbolic equation  
   first canonical form, 684  
   second canonical form, 684  
 hyperbolic equations  
   canonical form, 684  
   with one space variable, 191–273  
   with three space variables, 317–346  
   with two space variables, 275–316  
 hyperbolic system  
   conservation laws, 772  
   nonstrict, 786  
   quasilinear equations, 772, 774  
   strictly, 774

## I

ideal polytropic gas, equation of state, 772  
 infinitesimal operator, 736  
 infinitesimal transformation, 736  
 initial-boundary value problem, 773, 786  
 initial value problem, 773  
 integral equation, Gel'fand–Levitan–Marchenko, 519, 520, 523, 597, 606  
 invariance condition, 737  
 invariant of group, 737  
 invariant solutions, 694, 736, 741, 742  
 invariant surface condition, 744  
 inverse scattering method, 764

## K

Kadomtsev–Petviashvili equation, 605, 762  
   generalized, 629  
   canonical form, 605  
   unnormalized, 607  
 Kawahara's equation, 632  
 Khokhlov–Zabolotskaya equation, 435  
   generalized, 437  
   stationary, 353  
   three-dimensional, 438  
   two-dimensional, 435  
 Klein–Gordon equation, 234, 719, 721  
 Kolmogorov–Petrovskii–Piskunov equation, 71  
 Korteweg–de Vries equation, 515, 767  
   canonical form, 515  
   cylindrical, 521  
   generalized, 524  
   modified, 522  
   spherical, 522  
   unnormalized, 521  
 Kuramoto–Sivashinsky equation, 593

## L

Lagrangian, 770  
 Landau problem, 615  
 Laplace equation, 162, 173, 174, 182, 278  
 Laplace operator, 167, 170, 173, 190  
 law of conservation, 106, 108, 693, 769, 772  
 law of conservation of mass, 772, 777  
 law of conservation of momentum, 772, 777  
 Lax condition, 781  
 Lax pair, 764, 765  
 Lax pair method, 764  
 Legendre transformation, 263, 266, 404, 447, 689  
 Lie group methods, 735  
 Lie group of transformations, 736  
 limit self-similar solution, 696  
 linear subspaces invariant under nonlinear operator, 710, 711  
 linear wave equation, 684  
 Liouville equation, 213, 267  
   generalized, 679

## M

mathematical physics equations, *see equation and equations*  
 Maxwell's relations, 776  
 method  
   Clarkson–Kruskal direct, 728, 731  
   classical, symmetry reductions, 735  
   differential constraints, 747–749  
   direct, Clarkson–Kruskal, 728, 731  
   functional separation of variables, 713  
   generalized separation of variables, 698  
   group analysis, 735, 744, 757  
   inverse scattering, 764  
   Lax pair, 764, 765  
   nonclassical, for symmetry reductions, 744  
   similarity, 693, 695  
   splitting, 721  
   Titov–Galaktionov, 710  
 Mises transformation, 444, 543, 545, 552, 678  
 Miura transformation, 519, 523, 692  
 modified Burgers equation, 13  
 modified Korteweg–de Vries equation, 522  
 Monge–Ampère equation, 451  
   homogeneous, 451  
 movable pole, 760  
 movable singularities, 758  
 multiplicative separable solution, xviii, 698

## N

Newtonian fluid, 540, 553  
   steady hydrodynamic boundary layer equations, 540  
 Noetherian symmetries, 770  
 nonclassical method for symmetry reductions, 744  
 nonclassical symmetries, 744  
 nonlinear diffusion boundary layer equations, 124, 168, 169

nonlinear equations of the thermal boundary layer, 124  
 nonlinear functional equations, 723  
 nonlinear Klein–Gordon equation, 234  
 nonlinear Schrödinger equations, *see also Schrödinger equation*, 125, 186  
 nonlinear telegraph equations, 299–305  
 non-Newtonian fluids, 547  
   steady hydrodynamic boundary layer equations, 547  
 nonstationary equations, 576  
 nonstationary hydrodynamic equations, 574, 616  
 nonstrict hyperbolic system, 786  
 $N$ -soliton solution, 126, 228, 516, 605

## O

one-soliton solution, 126, 228, 515, 524, 599, 605  
 operator  
   infinitesimal, 736  
   Laplace, 167, 170, 173, 190  
   prolonged, 737, 771  
   total differential, 737, 750, 751, 753, 771

## P

Painlevé equation (ODE)  
   first, 518, 596, 759  
   second, 518, 523, 596, 759  
 Painlevé test, 758  
 parabolic equations  
   canonical form, 183  
   with one space variable, 1–140  
   with three space variables, 169–179  
   with two space variables, 141–168  
 partial differential equation, *see equation and equations*  
 point transformations, 685  
 Poisson equation, 150, 155, 163, 174, 184  
 problem  
   Blasius, 542  
   Blasius, generalized, 548  
   boundary layer, 118, 124  
   Cauchy, 10, 520, 446, 765, 773  
   Falkner–Skan, 544  
   Falkner–Skan, generalized, 550  
   initial-boundary value, 773, 774, 786  
   initial value, 773  
   Landau, 615  
   Riemann, 773, 777, 782  
   Schlichting, 542  
   Schlichting, generalized, 548  
 probability integral, 10  
 prolonged operator, 737, 771  
   coordinates of first prolongation, 737–740, 742, 745  
   coordinates of second prolongation, 737–740, 742, 745

## R

rarefaction waves, 777, 778  
 Rankine–Hugoniot jump conditions, 779  
 reductions, symmetry, 735  
 Riccati equation (ODE), 75, 77, 78, 81, 85  
 Riemann invariant, 776, 778  
 Riemann problem, 773, 777, 782

## S

Schlichting problem, 542  
   generalized, 548  
 Schrödinger equation,  
   of general form, 134, 135, 137, 138  
   three-dimensional, of general form, 189  
   three-dimensional, with cubic nonlinearity, 189  
   two-dimensional, of general form, 188  
   with cubic nonlinearity, 125, 127, 128, 129, 131, 189  
   with power-law nonlinearity, 127, 129  
 second-order differential constraints, 754  
 second-order equations of general form, 479–514, 754  
 second-order evolution equations, 749  
 second-order hyperbolic equations, 191–346, 753  
 self-similar solutions, xviii, 13, 24, 695, 777  
 separable equation (ODE), 35, 47, 208, 214, 366  
 separable solutions, generalized, xviii, 701, 709  
 shock (shock wave), 780, 781  
 similarity methods, 693  
 similarity reduction, 732, 734  
   special form, 728  
   general form, 731  
   generalized, 728, 757  
 single-soliton solution, *see also one-soliton solution*, 126, 228  
 sine-Gordon equation, 227, 229, 269, 766  
   double, 229  
 sinh-Gordon equation, 225, 268, 767  
   double, 225  
 soliton, 125, 228, 516, 521, 605  
 “soliton + pole” solution, 516  
 solution  
   additive separable, xviii, 698  
   Blasius, 125  
   exponential self-similar, 696  
   functional separable, xviii, 713  
   generalized separable, xviii, 701, 709  
   generalized traveling-wave, xviii, 713  
   invariant, 694, 736, 741, 742  
   limit self-similar, 696  
   multiplicative separable, xviii, 698  
    $N$ -soliton, 126, 228, 516, 605  
   one-soliton, 125, 228, 515, 524, 599, 605  
   Riemann problem, 782  
   self-similar, xviii, 13, 24, 695, 777  
   single-soliton, *see also one-soliton solution*, 126, 228  
   “soliton + pole”, 516  
   special functional separable, 713  
   traveling-wave, xviii, 5–8, 13, 14, 694

- traveling-wave, generalized, xviii, 5–8, 13, 36, 39, 713  
 two-soliton, 228, 516, 524, 599, 605  
 special functional separable solutions, 713  
 spherical Korteweg–de Vries equation, 522  
 splitting method, 705, 721  
 stationary anisotropic diffusion equation, 395  
 stationary anisotropic heat equation, 395  
 stationary heat equation with nonlinear source, 382  
 stationary hydrodynamic equations, 607  
 stationary Khokhlov–Zabolotskaya equation, 353  
 stationary Navier–Stokes equations, 607  
 steady hydrodynamic boundary layer equations  
   for Newtonian fluid, 540  
   for non-Newtonian fluids, 547  
 strictly hyperbolic system, 774  
 structural formula, 782  
 structure of functional separable solutions, 713  
 structure of generalized separable solutions, 700  
 symmetry reductions, 735  
 systems of conservation laws, 772
- T**
- telegraph equations  
   linear, 230, 285, 300, 301, 303  
   nonlinear, 299–305  
 third-order equations, 515–588  
 three-argument functional equations, 722  
 three-dimensional equations, 191  
 three-dimensional Khokhlov–Zabolotskaya equation, 438  
 three-dimensional nonlinear Schrödinger equation  
   of general form, 189  
 three-dimensional Schrödinger equation with cubic  
   nonlinearity, 189  
 Titov–Galaktionov method, 710  
 total differential operator, 737, 750, 751, 753, 771  
 transformation  
   auto-Bäcklund, 126, 691  
   Bäcklund, 11, 267, 519, 690–693  
   contact, 688  
   Crocco, 543, 546  
   Euler, 123, 447, 689  
   hodograph, 26, 122, 254, 356, 686  
   Hopf–Cole, 10, 692, 762  
   infinitesimal, 736  
   Legendre, 263, 266, 404, 447, 689  
   Mises, 444, 543, 545, 552, 678  
   Miura, 519, 523, 692  
   point, 685  
   traveling-wave solutions, xviii, 5–8, 13, 14, 694  
     generalized, xviii, 13, 36, 39, 713  
 two-dimensional Khokhlov–Zabolotskaya equation, 435  
 two-dimensional nonlinear Schrödinger equation  
   of general form, 188  
 two-dimensional Schrödinger equation with cubic  
   nonlinearity, 186  
 two-dimensional Schrödinger equation with power-  
   law nonlinearity, 187  
 two-soliton solution, 228, 516, 524, 599, 605  
 types of equations, 683  
 Tzitzéica equation, 268
- U**
- universal invariant of group, 737  
 unnormalized Boussinesq equation, 597  
 unnormalized Burgers equation, 11, 109  
 unnormalized Kadomtsev–Petviashvili equation, 607  
 unnormalized Korteweg–de Vries equation, 521  
 unsteady boundary layer equations for Newtonian  
   fluid, 555  
 unsteady boundary layer equations for non-New-  
   tonian fluids, 566
- V**
- vector Burgers equation, 179  
 von Mises transformation, 444, 543, 545, 552, 678
- W**
- wave  
   cnoidal, 515  
   nonlinear, 196, 198, 199, 201, 218, 239, 245  
   rarefaction, 777, 778  
   shock, 779  
 Weierstrass elliptic function, 2, 362, 533, 596, 598
- Y**
- Yermakov's equation (ODE), 97, 251