



CONTENTS

Authors xxi
Foreword xxiii
Notation and Some Remarks xxv
Introduction Some Definitions, Formulas, Methods, and Transformations 1
0.1. First-Order Differential Equations 1
0.1.1. General Concepts. The Cauchy Problem. Uniqueness and Existence Theorems 1
0.1.1-1. Equations solved for the derivative. General solution 1
0.1.1-2. The Cauchy problem. The uniqueness and existence theorems 1
0.1.1-3. Equations not solved for the derivative. The existence theorem 2
0.1.1-4. Singular solutions 2
0.1.1-5. Point transformations 2
0.1.2. Equations Solved for the Derivative. Simplest Techniques of Integration 3
0.1.2-1. Equations with separated or separable variables 3
0.1.2-2. Equation of the form y'_x = f(ax + by) 3
0.1.2-3. Homogeneous equations and equations reducible to them 3
0.1.2-4. Generalized homogeneous equations and equations reducible to them 4
0.1.2-5. Linear equation 4
0.1.2-6. Bernoulli equation 4
0.1.2-7. Equation of the form xy'_x = y + f(x)g(y/x) 5
0.1.2-8. Darboux equation 5
0.1.3. Exact Differential Equations. Integrating Factor 5
0.1.3-1. Exact differential equations 5
0.1.3-2. Integrating factor 5
0.1.4. Riccati Equation 6
0.1.4-1. General Riccati equation. Simplest integrable cases 6
0.1.4-2. Polynomial solutions of the Riccati equation 7
0.1.4-3. Use of particular solutions to construct the general solution 7
0.1.4-4. Some transformations 8
0.1.4-5. Reduction of the Riccati equation to a second-order linear equation 8
0.1.4-6. Reduction of the Riccati equation to the canonical form 8
0.1.5. Abel Equations of the First Kind 9
0.1.5-1. General form of Abel equations of the first kind. Simplest integrable cases 9
0.1.5-2. Reduction to the canonical form. Reduction to an Abel equation of the second kind 9
0.1.6. Abel Equations of the Second Kind 10
0.1.6-1. General form of Abel equations of the second kind. Simplest integrable cases 10
0.1.6-2. Reduction to the canonical form. Reduction to an Abel equation of the first kind 11
0.1.6-3. Use of particular solutions to construct self-transformations 11
0.1.6-4. Use of particular solutions to construct the general solution 12
0.1.7. Equations Not Solved for the Derivative 14
0.1.7-1. The method of "integration by differentiation." 14
0.1.7-2. Equations of the form y = f(y'_x) 14

0.1.7-3.	Equations of the form $x = f(y'_x)$	14
0.1.7-4.	Clairaut's equation $y = xy'_x + f(y'_x)$	15
0.1.7-5.	Lagrange's equation $y = xf(y'_x) + g(y'_x)$	15
0.1.8.	Contact Transformations	15
0.1.8-1.	General form of contact transformations	15
0.1.8-2.	A method for the construction of contact transformations	16
0.1.8-3.	Examples of contact transformations linear in the derivative	16
0.1.8-4.	Examples of contact transformations nonlinear in the derivative	17
0.1.9.	Approximate Analytic Methods for Solution of Equations	18
0.1.9-1.	The method of successive approximations (Picard method)	18
0.1.9-2.	The method of Taylor series expansion in the independent variable	18
0.1.9-3.	The method of regular expansion in the small parameter	19
0.1.10.	Numerical Integration of Differential Equations	20
0.1.10-1.	The method of Euler polygonal lines	20
0.1.10-2.	Single-step methods of the second-order approximation	20
0.1.10-3.	Runge–Kutta method of the fourth-order approximation	20
0.2.	Second-Order Linear Differential Equations	21
0.2.1.	Formulas for the General Solution. Some Transformations	21
0.2.1-1.	Homogeneous linear equations. Various representations of the general solution	21
0.2.1-2.	Wronskian determinant and Liouville's formula	21
0.2.1-3.	Reduction to the canonical form	21
0.2.1-4.	Reduction to the Riccati equation	22
0.2.1-5.	Nonhomogeneous linear equations. The existence theorem	22
0.2.1-6.	Nonhomogeneous linear equations. Various representations of the general solution	22
0.2.1-7.	Reduction to a constant coefficient equation (a special case)	22
0.2.1-8.	Kummer–Liouville transformation	23
0.2.2.	Representation of Solutions as a Series in the Independent Variable	23
0.2.2-1.	Equation coefficients are representable in the ordinary power series form	23
0.2.2-2.	Equation coefficients have poles at some point	23
0.2.3.	Asymptotic Solutions	24
0.2.3-1.	Equations not containing y'_x . Leading asymptotic terms	24
0.2.3-2.	Equations not containing y'_x . Two-term asymptotic expansions	25
0.2.3-3.	Equations of special form not containing y'_x	25
0.2.3-4.	Equations not containing y'_x . Equation coefficients are dependent on ε	26
0.2.3-5.	Equations containing y'_x	27
0.2.3-6.	Equations of the general form	27
0.2.4.	Boundary Value Problems	27
0.2.4-1.	The first, second, third, and mixed boundary value problems	27
0.2.4-2.	Simplification of boundary conditions. Reduction of equation to the self-adjoint form	28
0.2.4-3.	The Green's function. Boundary value problems for nonhomogeneous equations	28
0.2.4-4.	Representation of the Green's function in terms of particular solutions	29
0.2.5.	Eigenvalue Problems	29
0.2.5-1.	The Sturm–Liouville problem	29
0.2.5-2.	General properties of the Sturm–Liouville problem (1), (2)	29
0.2.5-3.	Problems with boundary conditions of the first kind	30
0.2.5-4.	Problems with boundary conditions of the second kind	32

0.2.5-5. Problems with boundary conditions of the third kind	33
0.2.5-6. Problems with mixed boundary conditions	33
0.3. Second-Order Nonlinear Differential Equations	33
0.3.1. Form of the General Solution. Cauchy Problem	33
0.3.1-1. Equations solved for the derivative. General solution	33
0.3.1-2. Cauchy problem. The existence and uniqueness theorem	34
0.3.2. Equations Admitting Reduction of Order	34
0.3.2-1. Equations not containing y explicitly	34
0.3.2-2. Equations not containing x explicitly (autonomous equations)	34
0.3.2-3. Equations of the form $F(ax + by, y'_x, y''_{xx}) = 0$	34
0.3.2-4. Equations of the form $F(x, xy'_x - y, y''_{xx}) = 0$	34
0.3.2-5. Homogeneous equations	35
0.3.2-6. Generalized homogeneous equations	35
0.3.2-7. Equations invariant under scaling–translation transformations	35
0.3.2-8. Exact second-order equations	36
0.3.2-9. Reduction of quasilinear equations to the normal form	37
0.3.3. Methods of Regular Series Expansions with Respect to the Independent Variable or Small Parameter	37
0.3.3-1. Method of expansion in powers of the independent variable	37
0.3.3-2. Method of regular (direct) expansion in powers of the small parameter	38
0.3.3-3. Padé approximants	39
0.3.4. Perturbation Methods of Mechanics and Physics	40
0.3.4-1. Preliminary remarks. A summary table of basic methods	40
0.3.4-2. The method of scaled parameters (Lindstedt–Poincaré method)	40
0.3.4-3. Averaging method (Van der Pol–Krylov–Bogolyubov scheme)	42
0.3.4-4. Method of two-scale expansions (Cole–Kevorkian scheme)	43
0.3.4-5. Method of matched asymptotic expansions	44
0.3.5. Galerkin Method and Its Modifications (Projection Methods)	46
0.3.5-1. General form of an approximate solution	46
0.3.5-2. Galerkin method	47
0.3.5-3. The Bubnov–Galerkin method, the moment method, and the least squares method	47
0.3.5-4. Collocation method	48
0.3.5-5. The method of partitioning the domain	48
0.3.5-6. The least squared error method	48
0.3.6. Iteration and Numerical Methods	49
0.3.6-1. The method of successive approximations (Cauchy problem)	49
0.3.6-2. The Runge–Kutta method (Cauchy problem)	49
0.3.6-3. Shooting method (boundary value problems)	49
0.3.6-4. Method of accelerated convergence in eigenvalue problems	50
0.4. Linear Equations of Arbitrary Order	51
0.4.1. Linear Equations with Constant Coefficients	51
0.4.1-1. Homogeneous linear equations	51
0.4.1-2. Nonhomogeneous linear equations	52
0.4.2. Linear Equations with Variable Coefficients	52
0.4.2-1. Homogeneous linear equations. Structure of the general solution	52
0.4.2-2. Utilization of particular solutions for reducing the order of the original equation	53
0.4.2-3. Wronskian determinant and Liouville formula	54

0.4.2-4. Nonhomogeneous linear equations. Construction of the general solution	54
0.4.3. Asymptotic Solutions of Linear Equations	54
0.4.3-1. Fourth-order linear equations	54
0.4.3-2. Higher-order linear equations	55
0.5. Nonlinear Equations of Arbitrary Order	56
0.5.1. Structure of the General Solution. Cauchy Problem	56
0.5.1-1. Equations solved for the highest derivative. General solution	56
0.5.1-2. The Cauchy problem. The existence and uniqueness theorem	56
0.5.2. Equations Admitting Reduction of Order	56
0.5.2-1. Equations not containing $y, y'_x, \dots, y_x^{(k)}$ explicitly	56
0.5.2-2. Equations not containing x explicitly (autonomous equations)	56
0.5.2-3. Equations of the form $F(ax + by, y'_x, \dots, y_x^{(n)}) = 0$	57
0.5.2-4. Equations of the form $F(x, xy'_x - y, y''_{xx}, \dots, y_x^{(n)}) = 0$ and its generalizations	57
0.5.2-5. Homogeneous equations	57
0.5.2-6. Generalized homogeneous equations	57
0.5.2-7. Equations of the form $F(e^{\lambda x} y^n, y'_x/y, y''_{xx}/y, \dots, y_x^{(n)}/y) = 0$	58
0.5.2-8. Equations of the form $F(x^n e^{\lambda y}, xy'_x, x^2 y''_{xx}, \dots, x^n y_x^{(n)}) = 0$	58
0.5.2-9. Other equations	58
0.5.3. A Method for Construction of Solvable Equations of General Form	59
0.5.3-1. Description of the method	59
0.5.3-2. Examples	59
0.6. Lie Group and Discrete-Group Methods	60
0.6.1. Lie Group Method. Point Transformations	60
0.6.1-1. Local one-parameter Lie group of transformations. Invariance condition	60
0.6.1-2. Group analysis of second-order equations. Structure of an admissible operator	62
0.6.1-3. Utilization of local groups for reducing the order of equations and their integration	64
0.6.2. Contact Transformations. Bäcklund Transformations. Formal Operators. Factorization Principle	65
0.6.2-1. Contact transformations	65
0.6.2-2. Bäcklund transformations. Formal operators and nonlocal variables	66
0.6.2-3. Factorization principle	68
0.6.3. First Integrals (Conservation Laws)	71
0.6.4. Discrete-Group Method. Point Transformations	73
0.6.5. Discrete-Group Method. The Method of RF-Pairs	75
1. First-Order Differential Equations	81
1.1. Simplest Equations with Arbitrary Functions Integrable in Closed Form	81
1.1.1. Equations of the Form $y'_x = f(x)$	81
1.1.2. Equations of the Form $y'_x = f(y)$	81
1.1.3. Separable Equations $y'_x = f(x)g(y)$	81
1.1.4. Linear Equation $g(x)y'_x = f_1(x)y + f_0(x)$	81
1.1.5. Bernoulli Equation $g(x)y'_x = f_1(x)y + f_n(x)y^n$	81
1.1.6. Homogeneous Equation $y'_x = f(y/x)$	82

1.2. Riccati Equation $g(x)y'_x = f_2(x)y^2 + f_1(x)y + f_0(x)$	82
1.2.1. Preliminary Remarks	82
1.2.2. Equations Containing Power Functions	82
1.2.2-1. Equations of the form $g(x)y'_x = f_2(x)y^2 + f_0(x)$	82
1.2.2-2. Other equations	84
1.2.3. Equations Containing Exponential Functions	89
1.2.3-1. Equations with exponential functions	89
1.2.3-2. Equations with power and exponential functions	90
1.2.4. Equations Containing Hyperbolic Functions	92
1.2.4-1. Equations with hyperbolic sine and cosine	92
1.2.4-2. Equations with hyperbolic tangent and cotangent	93
1.2.5. Equations Containing Logarithmic Functions	94
1.2.5-1. Equations of the form $g(x)y'_x = f_2(x)y^2 + f_0(x)$	94
1.2.5-2. Equations of the form $g(x)y'_x = f_2(x)y^2 + f_1(x)y'_x + f_0(x)$	94
1.2.6. Equations Containing Trigonometric Functions	95
1.2.6-1. Equations with sine	95
1.2.6-2. Equations with cosine	96
1.2.6-3. Equations with tangent	97
1.2.6-4. Equations with cotangent	98
1.2.6-5. Equations containing combinations of trigonometric functions	99
1.2.7. Equations Containing Inverse Trigonometric Functions	100
1.2.7-1. Equations containing arcsine	100
1.2.7-2. Equations containing arccosine	100
1.2.7-3. Equations containing arctangent	101
1.2.7-4. Equations containing arccotangent	101
1.2.8. Equations with Arbitrary Functions	102
1.2.8-1. Equations containing arbitrary functions (but not containing their derivatives)	102
1.2.8-2. Equations containing arbitrary functions and their derivatives	104
1.2.9. Some Transformations	105
1.3. Abel Equations of the Second Kind	107
1.3.1. Equations of the Form $yy'_x - y = f(x)$	107
1.3.1-1. Preliminary remarks. Classification tables	107
1.3.1-2. Solvable equations and their solutions	109
1.3.2. Equations of the Form $yy'_x = f(x)y + 1$	120
1.3.3. Equations of the Form $yy'_x = f_1(x)y + f_0(x)$	121
1.3.3-1. Preliminary remarks	121
1.3.3-2. Solvable equations and their solutions	121
1.3.4. Equations of the Form $[g_1(x)y + g_0(x)]y'_x = f_2(x)y^2 + f_1(x)y + f_0(x)$	132
1.3.4-1. Preliminary remarks	132
1.3.4-2. Solvable equations and their solutions	132
1.3.5. Some Types of First- and Second-Order Equations Reducible to Abel Equations of the Second Kind	136
1.3.5-1. Quasi-homogeneous equations	136
1.3.5-2. Equations of the theory of chemical reactors and the combustion theory	136
1.3.5-3. Equations of the theory of nonlinear oscillations	136
1.3.5-4. Second-order homogeneous equations of various types	137
1.3.5-5. Second-order equations invariant under some transformations	137

1.4. Equations Containing Polynomial Functions of y	138
1.4.1. Abel Equations of the First Kind $y'_x = f_3(x)y^3 + f_2(x)y^2 + f_1(x)y + f_0(x)$	138
1.4.1-1. Preliminary remarks	138
1.4.1-2. Solvable equations and their solutions	138
1.4.2. Equations of the Form $(A_{22}y^2 + A_{12}xy + A_{11}x^2 + A_0)y'_x = B_{22}y^2 + B_{12}xy + B_{11}x^2 + B_0$	142
1.4.2-1. Preliminary remarks. Some transformations	142
1.4.2-2. Solvable equations and their solutions	143
1.4.3. Equations of the Form $(A_{22}y^2 + A_{12}xy + A_{11}x^2 + A_2y + A_1x)y'_x = B_{22}y^2 + B_{12}xy + B_{11}x^2 + B_2y + B_1x$	144
1.4.3-1. Preliminary remarks	144
1.4.3-2. Solvable equations and their solutions	145
1.4.4. Equations of the Form $(A_{22}y^2 + A_{12}xy + A_{11}x^2 + A_2y + A_1x + A_0)y'_x = B_{22}y^2 + B_{12}xy + B_{11}x^2 + B_2y + B_1x + B_0$	151
1.4.4-1. Preliminary remarks. Some transformations	151
1.4.4-2. Solvable equations and their solutions	152
1.4.5. Equations of the Form $(A_3y^3 + A_2xy^2 + A_1x^2y + A_0x^3 + a_1y + a_0x)y'_x = B_3y^3 + B_2xy^2 + B_1x^2y + B_0x^3 + b_1y + b_0x$	155
1.5. Equations of the Form $f(x, y)y'_x = g(x, y)$ Containing Arbitrary Parameters	159
1.5.1. Equations Containing Power Functions	159
1.5.1-1. Equations of the form $y'_x = f(x, y)$	159
1.5.1-2. Other equations	160
1.5.2. Equations Containing Exponential Functions	162
1.5.2-1. Equations with exponential functions	162
1.5.2-2. Equations with power and exponential functions	163
1.5.3. Equations Containing Hyperbolic Functions	166
1.5.4. Equations Containing Logarithmic Functions	168
1.5.5. Equations Containing Trigonometric Functions	169
1.5.6. Equations Containing Combinations of Exponential, Hyperbolic, Logarithmic, and Trigonometric Functions	171
1.6. Equations of the Form $F(x, y, y'_x) = 0$ Containing Arbitrary Parameters	173
1.6.1. Equations of the Second Degree in y'_x	173
1.6.1-1. Equations of the form $f(x, y)(y'_x)^2 = g(x, y)$	173
1.6.1-2. Equations of the form $f(x, y)(y'_x)^2 = g(x, y)y'_x + h(x, y)$	175
1.6.2. Equations of the Third Degree in y'_x	180
1.6.2-1. Equations of the form $f(x, y)(y'_x)^3 = g(x, y)y'_x + h(x, y)$	180
1.6.2-2. Equations of the form $f(x, y)(y'_x)^3 = g(x, y)(y'_x)^2 + h(x, y)y'_x + r(x, y)$	180
1.6.3. Equations of the Form $(y'_x)^k = f(y) + g(x)$	182
1.6.3-1. Some transformations	182
1.6.3-2. Classification tables and exact solutions	183
1.6.4. Other Equations	190
1.6.4-1. Equations containing algebraic and power functions with respect to y'_x	190
1.6.4-2. Equations containing exponential, logarithmic, and other functions with respect to y'_x	193
1.7. Equations of the Form $f(x, y)y'_x = g(x, y)$ Containing Arbitrary Functions	195
1.7.1. Equations Containing Power Functions	195
1.7.2. Equations Containing Exponential and Hyperbolic Functions	197
1.7.3. Equations Containing Logarithmic Functions	199
1.7.4. Equations Containing Trigonometric Functions	200
1.7.5. Equations Containing Combinations of Exponential, Logarithmic, and Trigonometric Functions	201

1.8. Equations of the Form $F(x, y, y'_x) = 0$ Containing Arbitrary Functions	203
1.8.1. Some Equations	203
1.8.1-1. Arguments of arbitrary functions depend on x and y	203
1.8.1-2. Argument of arbitrary functions is y'_x	204
1.8.1-3. Arguments of arbitrary functions are linear with respect to y'_x	205
1.8.1-4. Arguments of arbitrary functions are nonlinear with respect to y'_x	209
1.8.2. Some Transformations	212
2. Second-Order Differential Equations	213
2.1. Linear Equations	213
2.1.1. Representation of the General Solution Through a Particular Solution	213
2.1.2. Equations Containing Power Functions	213
2.1.2-1. Equations of the form $y''_{xx} + f(x)y = 0$	213
2.1.2-2. Equations of the form $y''_{xx} + f(x)y'_x + g(x)y = 0$	215
2.1.2-3. Equations of the form $(ax + b)y''_{xx} + f(x)y'_x + g(x)y = 0$	219
2.1.2-4. Equations of the form $x^2y''_{xx} + f(x)y'_x + g(x)y = 0$	225
2.1.2-5. Equations of the form $(ax^2 + bx + c)y''_{xx} + f(x)y'_x + g(x)y = 0$	230
2.1.2-6. Equations of the form $(a_3x^3 + a_2x^2 + a_1x + a_0)y''_{xx} + f(x)y'_x + g(x)y = 0$	237
2.1.2-7. Equations of the form $(a_4x^4 + \dots + a_1x + a_0)y''_{xx} + f(x)y'_x + g(x)y = 0$	240
2.1.2-8. Other equations	244
2.1.3. Equations Containing Exponential Functions	246
2.1.3-1. Equations with exponential functions	246
2.1.3-2. Equations with power and exponential functions	250
2.1.4. Equations Containing Hyperbolic Functions	252
2.1.4-1. Equations with hyperbolic sine	252
2.1.4-2. Equations with hyperbolic cosine	253
2.1.4-3. Equations with hyperbolic tangent	254
2.1.4-4. Equations with hyperbolic cotangent	255
2.1.4-5. Equations containing combinations of hyperbolic functions	256
2.1.5. Equations Containing Logarithmic Functions	257
2.1.5-1. Equations of the form $f(x)y''_{xx} + g(x)y = 0$	257
2.1.5-2. Equations of the form $f(x)y''_{xx} + g(x)y'_x + h(x)y = 0$	258
2.1.6. Equations Containing Trigonometric Functions	260
2.1.6-1. Equations with sine	260
2.1.6-2. Equations with cosine	262
2.1.6-3. Equations with tangent	265
2.1.6-4. Equations with cotangent	267
2.1.6-5. Equations containing combinations of trigonometric functions	269
2.1.7. Equations Containing Inverse Trigonometric Functions	271
2.1.7-1. Equations with arcsine	271
2.1.7-2. Equations with arccosine	273
2.1.7-3. Equations with arctangent	274
2.1.7-4. Equations with arccotangent	276
2.1.8. Equations Containing Combinations of Exponential, Logarithmic, Trigonometric, and Other Functions	277
2.1.9. Equations with Arbitrary Functions	285
2.1.9-1. Equations containing arbitrary functions (but not containing their derivatives)	285
2.1.9-2. Equations containing arbitrary functions and their derivatives	289
2.1.10. Some Transformations	292

2.2. Autonomous Equations $y''_{xx} = F(y, y'_x)$	295
2.2.1. Equations of the Form $y''_{xx} - y'_x = f(y)$	295
2.2.2. Equations of the Form $y''_{xx} + f(y)y'_x + y = 0$	299
2.2.2-1. Preliminary remarks	299
2.2.2-2. Solvable equations and their solutions	299
2.2.3. Lienard Equations $y''_{xx} + f(y)y'_x + g(y) = 0$	301
2.2.3-1. Preliminary remarks	301
2.2.3-2. Solvable equations and their solutions	302
2.2.4. Rayleigh Equations $y''_{xx} + f(y'_x) + g(y) = 0$	304
2.2.4-1. Preliminary remarks. Some transformations	304
2.2.4-2. Solvable equations and their solutions	305
2.3. Emden–Fowler Equation $y''_{xx} = Ax^n y^m$	306
2.3.1. Exact Solutions	306
2.3.1-1. Preliminary remarks. Classification table	306
2.3.1-2. Solvable equations and their solutions	307
2.3.2. First Integrals (Conservation Laws)	312
2.3.2-1. First integrals with $k = 2$	312
2.3.2-2. First integrals with $k = 3$	312
2.3.2-3. First integrals with $k = 4$	313
2.3.2-4. First integrals with $k = 5$	314
2.3.3. Some Formulas and Transformations	314
2.4. Equations of the Form $y''_{xx} = A_1 x^{n_1} y^{m_1} + A_2 x^{n_2} y^{m_2}$	314
2.4.1. Classification Table	314
2.4.2. Exact Solutions	318
2.5. Generalized Emden–Fowler Equation $y''_{xx} = Ax^n y^m (y'_x)^l$	336
2.5.1. Classification Table	336
2.5.2. Exact Solutions	339
2.5.3. Some Formulas and Transformations	354
2.5.3-1. A particular solution	354
2.5.3-2. Discrete transformations of the generalized Emden–Fowler equation	355
2.5.3-3. Reduction of the generalized Emden–Fowler equation to an Abel equation	355
2.6. Equations of the Form $y''_{xx} = A_1 x^{n_1} y^{m_1} (y'_x)^{l_1} + A_2 x^{n_2} y^{m_2} (y'_x)^{l_2}$	356
2.6.1. Modified Emden–Fowler Equation $y''_{xx} = A_1 x^{-1} y'_x + A_2 x^n y^m$	356
2.6.1-1. Preliminary remarks. Classification table	356
2.6.1-2. Solvable equations and their solutions	358
2.6.2. Equations of the Form $y''_{xx} = (A_1 x^{n_1} y^{m_1} + A_2 x^{n_2} y^{m_2})(y'_x)^l$	365
2.6.2-1. Classification table	365
2.6.2-2. Solvable equations and their solutions	370
2.6.3. Equations of the Form $y''_{xx} = \sigma A x^n y^m (y'_x)^l + A x^{n-1} y^{m+1} (y'_x)^{l-1}$	393
2.6.3-1. Classification table	393
2.6.3-2. Solvable equations and their solutions	396
2.6.4. Other Equations ($l_1 \neq l_2$)	406
2.6.4-1. Classification table	406
2.6.4-2. Solvable equations and their solutions	407
2.7. Equations of the Form $y''_{xx} = f(x)g(y)h(y'_x)$	411
2.7.1. Equations of the Form $y''_{xx} = f(x)g(y)$	412
2.7.2. Equations Containing Power Functions ($h \neq \text{const}$)	414
2.7.3. Equations Containing Exponential Functions ($h \neq \text{const}$)	418

2.7.3-1. Preliminary remarks	418
2.7.3-2. Solvable equations and their solutions	419
2.7.4. Equations Containing Hyperbolic Functions ($h \neq \text{const}$)	421
2.7.5. Equations Containing Trigonometric Functions ($h \neq \text{const}$)	423
2.7.6. Some Transformations	424
2.8. Some Nonlinear Equations with Arbitrary Parameters	425
2.8.1. Equations Containing Power Functions	425
2.8.1-1. Equations of the form $f(x, y)y''_{xx} + g(x, y) = 0$	425
2.8.1-2. Equations of the form $f(x, y)y''_{xx} + g(x, y)y'_x + h(x, y) = 0$	427
2.8.1-3. Equations of the form $f(x, y)y''_{xx} + g(x, y)(y'_x)^2 + h(x, y)y'_x + r(x, y) = 0$	428
2.8.1-4. Other equations	430
2.8.2. Painlevé Transcendents	432
2.8.2-1. Preliminary remarks. Singular points of solutions	432
2.8.2-2. First Painlevé transcendent	432
2.8.2-3. Second Painlevé transcendent	433
2.8.2-4. Third Painlevé transcendent	434
2.8.2-5. Fourth Painlevé transcendent	435
2.8.2-6. Fifth Painlevé transcendent	436
2.8.2-7. Sixth Painlevé transcendent	437
2.8.3. Equations Containing Exponential Functions	438
2.8.3-1. Equations of the form $f(x, y)y''_{xx} + g(x, y) = 0$	438
2.8.3-2. Equations of the form $f(x, y)y''_{xx} + g(x, y)y'_x + h(x, y) = 0$	438
2.8.3-3. Equations of the form $f(x, y)y''_{xx} + g(x, y)(y'_x)^2 + h(x, y)y'_x + r(x, y) = 0$	441
2.8.3-4. Other equations	443
2.8.4. Equations Containing Hyperbolic Functions	445
2.8.4-1. Equations with hyperbolic sine	445
2.8.4-2. Equations with hyperbolic cosine	447
2.8.4-3. Equations with hyperbolic tangent	448
2.8.4-4. Equations with hyperbolic cotangent	449
2.8.4-5. Equations containing combinations of hyperbolic functions	450
2.8.5. Equations Containing Logarithmic Functions	450
2.8.5-1. Equations of the form $f(x, y)y''_{xx} + g(x, y)y'_x + h(x, y) = 0$	450
2.8.5-2. Other equations	451
2.8.6. Equations Containing Trigonometric Functions	452
2.8.6-1. Equations with sine	452
2.8.6-2. Equations with cosine	454
2.8.6-3. Equations with tangent	456
2.8.6-4. Equations with cotangent	457
2.8.6-5. Equations containing combinations of trigonometric functions	458
2.8.7. Equations Containing the Combinations of Exponential, Hyperbolic, Logarithmic, and Trigonometric Functions	459
2.9. Equations Containing Arbitrary Functions	461
2.9.1. Equations of the Form $F(x, y)y''_{xx} + G(x, y) = 0$	461
2.9.1-1. Arguments of arbitrary functions are algebraic and power functions of x and y	461
2.9.1-2. Arguments of the arbitrary functions are other functions	465
2.9.2. Equations of the Form $F(x, y)y''_{xx} + G(x, y)y'_x + H(x, y) = 0$	467
2.9.2-1. Argument of the arbitrary functions is x	467
2.9.2-2. Argument of the arbitrary functions is y	468
2.9.2-3. Other arguments of the arbitrary functions	469

2.9.3.	Equations of the Form $F(x, y)y''_{xx} + \sum_{m=0}^M G_m(x, y)(y'_x)^m = 0$ ($M = 2, 3, 4$)	471
2.9.3-1.	Argument of the arbitrary functions is x	471
2.9.3-2.	Argument of the arbitrary functions is y	473
2.9.3-3.	Other arguments of arbitrary functions	474
2.9.4.	Equations of the Form $F(x, y, y'_x)y''_{xx} + G(x, y, y'_x) = 0$	475
2.9.4-1.	Arguments of the arbitrary functions depend on x or y	475
2.9.4-2.	Arguments of the arbitrary functions depend on x and y	476
2.9.4-3.	Arguments of the arbitrary functions depend on x, y , and y'_x	479
2.9.5.	Equations Not Solved for Second Derivative	484
2.9.6.	Equations of General Form	486
2.9.6-1.	Equations containing arbitrary functions of two variables	486
2.9.6-2.	Equations containing arbitrary functions of three variables	491
2.9.7.	Some Transformations	492
3.	Third-Order Differential Equations	495
3.1.	Linear Equations	495
3.1.1.	Preliminary Remarks	495
3.1.2.	Equations Containing Power Functions	496
3.1.2-1.	Equations of the form $f_3(x)y'''_{xxx} + f_0(x)y = g(x)$	496
3.1.2-2.	Equations of the form $f_3(x)y'''_{xxx} + f_1(x)y'_x + f_0(x)y = g(x)$	497
3.1.2-3.	Equations of the form $f_3(x)y'''_{xxx} + f_2(x)y''_{xx} + f_1(x)y'_x + f_0(x)y = g(x)$	503
3.1.3.	Equations Containing Exponential Functions	512
3.1.3-1.	Equations with exponential functions	512
3.1.3-2.	Equations with power and exponential functions	515
3.1.4.	Equations Containing Hyperbolic Functions	516
3.1.4-1.	Equations with hyperbolic sine	516
3.1.4-2.	Equations with hyperbolic cosine	518
3.1.4-3.	Equations with hyperbolic sine and cosine	520
3.1.4-4.	Equations with hyperbolic tangent	520
3.1.4-5.	Equations with hyperbolic cotangent	523
3.1.5.	Equations Containing Logarithmic Functions	525
3.1.5-1.	Equations with logarithmic functions	525
3.1.5-2.	Equations with power and logarithmic functions	526
3.1.6.	Equations Containing Trigonometric Functions	528
3.1.6-1.	Equations with sine	528
3.1.6-2.	Equations with cosine	531
3.1.6-3.	Equations with sine and cosine	533
3.1.6-4.	Equations with tangent	534
3.1.6-5.	Equations with cotangent	537
3.1.7.	Equations Containing Inverse Trigonometric Functions	539
3.1.8.	Equations Containing Combinations of Exponential, Logarithmic, Trigonometric, and Other Functions	544
3.1.9.	Equations Containing Arbitrary Functions	550
3.1.9-1.	Equations of the form $f_3(x)y'''_{xxx} + f_1(x)y'_x + f_0(x)y = g(x)$	550
3.1.9-2.	Equations of the form $f_3(x)y'''_{xxx} + f_2(x)y''_{xx} + f_1(x)y'_x + f_0(x)y = g(x)$	553
3.2.	Equations of the Form $y'''_{xxx} = Ax^\alpha y^\beta (y'_x)^\gamma (y''_{xx})^\delta$	559
3.2.1.	Classification Table	559
3.2.2.	Equations of the Form $y'''_{xxx} = Ay^\beta$	566

3.2.3. Equations of the Form $y'''_{xxx} = Ax^\alpha y^\beta$	567
3.2.4. Equations with $ \gamma + \delta \neq 0$	568
3.2.5. Some Transformations	592
3.3. Equations of the Form $y'''_{xxx} = f(y)g(y'_x)h(y''_{xx})$	592
3.3.1. Equations Containing Power Functions	592
3.3.2. Equations Containing Exponential Functions	595
3.3.3. Other Equations	599
3.4. Nonlinear Equations with Arbitrary Parameters	601
3.4.1. Equations Containing Power Functions	601
3.4.1-1. Equations of the form $f(x, y)y'''_{xxx} = g(x, y)$	601
3.4.1-2. Equations of the form $y'''_{xxx} = f(x, y, y'_x)$	602
3.4.1-3. Equations of the form $f(x, y, y'_x)y'''_{xxx} + g(x, y, y'_x)y''_{xx} + h(x, y, y'_x) = 0$	603
3.4.1-4. Other equations	607
3.4.2. Equations Containing Exponential Functions	608
3.4.2-1. Equations of the form $y'''_{xxx} = f(x, y, y'_x)$	608
3.4.2-2. Other equations	609
3.4.3. Equations Containing Hyperbolic Functions	611
3.4.3-1. Equations with hyperbolic sine	611
3.4.3-2. Equations with hyperbolic cosine	612
3.4.3-3. Equations with hyperbolic tangent	613
3.4.3-4. Equations with hyperbolic cotangent	615
3.4.4. Equations Containing Logarithmic Functions	616
3.4.4-1. Equations of the form $y'''_{xxx} = f(x, y, y'_x)$	616
3.4.4-2. Other equations	617
3.4.5. Equations Containing Trigonometric Functions	618
3.4.5-1. Equations with sine	618
3.4.5-2. Equations with cosine	619
3.4.5-3. Equations with tangent	620
3.4.5-4. Equations with cotangent	621
3.5. Nonlinear Equations Containing Arbitrary Functions	622
3.5.1. Equations of the Form $F(x, y)y'''_{xxx} + G(x, y) = 0$	622
3.5.1-1. Arguments of the arbitrary functions are x or y	622
3.5.1-2. Arguments of the arbitrary functions depend on x and y	623
3.5.2. Equations of the Form $F(x, y, y'_x)y'''_{xxx} + G(x, y, y'_x) = 0$	624
3.5.2-1. Arguments of the arbitrary functions depend on x and y	624
3.5.2-2. Arguments of the arbitrary functions depend on $x, y,$ and y'_x	625
3.5.3. Equations of the Form $F(x, y, y'_x)y'''_{xxx} + G(x, y, y'_x)y''_{xx} + H(x, y, y'_x) = 0$	629
3.5.3-1. The arbitrary functions depend on x or y	629
3.5.3-2. Arguments of arbitrary functions depend on x and y	632
3.5.3-3. Arguments of arbitrary functions depend on $x, y,$ and y'_x	633
3.5.4. Equations of the Form $F(x, y, y'_x)y'''_{xxx} + \sum_{\alpha} G_{\alpha}(x, y, y'_x)(y''_{xx})^{\alpha} = 0$	634
3.5.4-1. Arbitrary functions depend on x or y	634
3.5.4-2. Arguments of arbitrary functions depend on $x, y,$ and y'_x	635
3.5.5. Other Equations	636
3.5.5-1. Equations of the form $F(x, y, y'_x, y''_{xx})y'''_{xxx} + G(x, y, y'_x, y''_{xx}) = 0$	636
3.5.5-2. Equations of the form $F(x, y, y'_x, y''_{xx}, y'''_{xxx}) = 0$	638

4. Fourth-Order Differential Equations	641
4.1. Linear Equations	641
4.1.1. Preliminary Remarks	641
4.1.2. Equations Containing Power Functions	641
4.1.2-1. Equations of the form $f_4(x)y'''' + f_0(x)y = g(x)$	641
4.1.2-2. Equations of the form $f_4(x)y'''' + f_1(x)y'_x + f_0(x)y = g(x)$	642
4.1.2-3. Equations of the form $f_4(x)y'''' + f_2(x)y''_{xx} + f_1(x)y'_x + f_0(x)y = g(x)$	644
4.1.2-4. Other equations	645
4.1.3. Equations Containing Exponential and Hyperbolic Functions	648
4.1.3-1. Equations with exponential functions	648
4.1.3-2. Equations with hyperbolic functions	649
4.1.4. Equations Containing Logarithmic Functions	651
4.1.5. Equations Containing Trigonometric Functions	652
4.1.5-1. Equations with sine and cosine	652
4.1.5-2. Equations with tangent and cotangent	654
4.1.6. Equations Containing Arbitrary Functions	655
4.1.6-1. Equations of the form $f_4(x)y'''' + f_1(x)y'_x + f_0(x)y = g(x)$	655
4.1.6-2. Equations of the form $f_4(x)y'''' + f_2(x)y''_{xx} + f_1(x)y'_x + f_0(x)y = g(x)$	657
4.1.6-3. Other equations	657
4.2. Nonlinear Equations	659
4.2.1. Equations Containing Power Functions	659
4.2.1-1. Equations of the form $y'''' = f(x, y)$	659
4.2.1-2. Equations of the form $y'''' = f(x, y, y'_x)$	660
4.2.1-3. Equations of the form $y'''' = f(x, y, y'_x, y''_{xx})$	660
4.2.1-4. Equations of the form $y'''' = f(x, y, y'_x, y''_{xx}, y'''_{xxx})$	663
4.2.2. Equations Containing Exponential Functions	666
4.2.2-1. Equations of the form $y'''' = f(x, y)$	666
4.2.2-2. Other equations	666
4.2.3. Equations Containing Hyperbolic Functions	668
4.2.3-1. Equations with hyperbolic sine	668
4.2.3-2. Equations with hyperbolic cosine	669
4.2.3-3. Equations with hyperbolic tangent	671
4.2.3-4. Equations with hyperbolic cotangent	671
4.2.4. Equations Containing Logarithmic Functions	672
4.2.4-1. Equations of the form $y'''' = f(x, y)$	672
4.2.4-2. Other equations	673
4.2.5. Equations Containing Trigonometric Functions	674
4.2.5-1. Equations with sine	674
4.2.5-2. Equations with cosine	675
4.2.5-3. Equations with tangent	677
4.2.5-4. Equations with cotangent	677
4.2.6. Equations Containing Arbitrary Functions	678
4.2.6-1. Equations of the form $y'''' = f(x, y)$	678
4.2.6-2. Equations of the form $y'''' = f(x, y, y'_x)$	680
4.2.6-3. Equations of the form $y'''' = f(x, y, y'_x, y''_{xx})$	681
4.2.6-4. Equations of the form $y'''' = f(x, y, y'_x, y''_{xx}, y'''_{xxx})$	683
4.2.6-5. Other equations	686

5. Higher-Order Differential Equations	689
5.1. Linear Equations	689
5.1.1. Preliminary Remarks	689
5.1.2. Equations Containing Power Functions	689
5.1.2-1. Equations of the form $f_n(x)y_x^{(n)} + f_0(x)y = g(x)$	689
5.1.2-2. Equations of the form $f_n(x)y_x^{(n)} + f_1(x)y'_x + f_0(x)y = g(x)$	692
5.1.2-3. Other equations	692
5.1.3. Equations Containing Exponential and Hyperbolic Functions	695
5.1.3-1. Equations with exponential functions	695
5.1.3-2. Equations with hyperbolic functions	696
5.1.4. Equations Containing Logarithmic Functions	698
5.1.5. Equations Containing Trigonometric Functions	698
5.1.5-1. Equations with sine and cosine	698
5.1.5-2. Equations with tangent and cotangent	700
5.1.6. Equations Containing Arbitrary Functions	701
5.1.6-1. Equations of the form $f_n(x)y_x^{(n)} + f_1(x)y'_x + f_0(x)y = g(x)$	701
5.1.6-2. Other equations	702
5.2. Nonlinear Equations	705
5.2.1. Equations Containing Power Functions	705
5.2.1-1. Fifth- and sixth-order equations	705
5.2.1-2. Equations of the form $y_x^{(n)} = f(x, y)$	706
5.2.1-3. Equations of the form $y_x^{(n)} = f(x, y, y'_x, y''_{xx})$	707
5.2.1-4. Other equations	708
5.2.2. Equations Containing Exponential Functions	711
5.2.2-1. Fifth- and sixth-order equations	711
5.2.2-2. Equations of the form $y_x^{(n)} = f(x, y)$	712
5.2.2-3. Other equations	712
5.2.3. Equations Containing Hyperbolic Functions	713
5.2.3-1. Equations with hyperbolic sine	713
5.2.3-2. Equations with hyperbolic cosine	714
5.2.3-3. Equations with hyperbolic tangent	715
5.2.3-4. Equations with hyperbolic cotangent	716
5.2.4. Equations Containing Logarithmic Functions	717
5.2.4-1. Equations of the form $y_x^{(n)} = f(x, y)$	717
5.2.4-2. Other equations	718
5.2.5. Equations Containing Trigonometric Functions	718
5.2.5-1. Equations with sine	718
5.2.5-2. Equations with cosine	719
5.2.5-3. Equations with tangent	720
5.2.5-4. Equations with cotangent	721
5.2.6. Equations Containing Arbitrary Functions	722
5.2.6-1. Fifth- and sixth-order equations	722
5.2.6-2. Equations of the form $y_x^{(n)} = f(x, y)$	723
5.2.6-3. Equations of the form $y_x^{(n)} = f(x, y, y'_x)$	726
5.2.6-4. Equations of the form $y_x^{(n)} = f(x, y, y'_x, y''_{xx})$	727
5.2.6-5. Equations of the form $f(x, y)y_x^{(n)} + g(x, y, y'_x)y_x^{(n-1)} =$ $h(x, y, y'_x, \dots, y_x^{(n-2)})$	728
5.2.6-6. Equations of the form $y_x^{(n)} = f(x, y, y'_x, \dots, y_x^{(n-1)})$	730
5.2.6-7. Equations of the general form $F(x, y, y'_x, \dots, y_x^{(n)}) = 0$	731

Supplements	735
S.1. Elementary Functions and Their Properties	735
S.1.1. Trigonometric Functions	735
S.1.1-1. Simplest relations	735
S.1.1-2. Relations between trigonometric functions of single argument	735
S.1.1-3. Reduction formulas	735
S.1.1-4. Addition and subtraction of trigonometric functions	736
S.1.1-5. Products of trigonometric functions	736
S.1.1-6. Powers of trigonometric functions	736
S.1.1-7. Addition formulas	737
S.1.1-8. Trigonometric functions of multiple arguments	737
S.1.1-9. Trigonometric functions of half argument	737
S.1.1-10. Euler and de Moivre formulas. Relationship with hyperbolic functions	737
S.1.1-11. Differentiation formulas	737
S.1.1-12. Expansion into power series	738
S.1.2. Hyperbolic Functions	738
S.1.2-1. Definitions	738
S.1.2-2. Simplest relations	738
S.1.2-3. Relations between hyperbolic functions of single argument ($x \geq 0$) ..	738
S.1.2-4. Addition formulas	738
S.1.2-5. Addition and subtraction of hyperbolic functions	739
S.1.2-6. Products of hyperbolic functions	739
S.1.2-7. Powers of hyperbolic functions	739
S.1.2-8. Hyperbolic functions of multiple arguments	739
S.1.2-9. Relationship with trigonometric functions	740
S.1.2-10. Differentiation formulas	740
S.1.2-11. Expansion into power series	740
S.1.3. Inverse Trigonometric Functions	740
S.1.3-1. Definitions and some properties	740
S.1.3-2. Simplest formulas	741
S.1.3-3. Relations between inverse trigonometric functions	741
S.1.3-4. Addition and subtraction of inverse trigonometric functions	741
S.1.3-5. Differentiation formulas	741
S.1.3-6. Expansion into power series	742
S.1.4. Inverse Hyperbolic Functions	742
S.1.4-1. Relationships with logarithmic functions	742
S.1.4-2. Relations between inverse hyperbolic functions	742
S.1.4-3. Addition and subtraction of inverse hyperbolic functions	742
S.1.4-4. Differentiation formulas	742
S.1.4-5. Expansion into power series	743
S.2. Special Functions and Their Properties	743
S.2.1. Some Symbols and Coefficients	743
S.2.1-1. Factorials	743
S.2.1-2. Binomial coefficients	743
S.2.1-3. Pochhammer symbol	744
S.2.2. Error Functions and Exponential Integral	744
S.2.2-1. Error function and complementary error function	744
S.2.2-2. Exponential integral	744
S.2.2-3. Logarithmic integral	745

S.2.3. Gamma and Beta Functions	745
S.2.3-1. Gamma function	745
S.2.3-2. Logarithmic derivative of the gamma function	746
S.2.3-3. Beta function	747
S.2.4. Incomplete Gamma and Beta Functions	747
S.2.4-1. Incomplete gamma function	747
S.2.4-2. Incomplete beta function	747
S.2.5. Bessel Functions	748
S.2.5-1. Definitions and basic formulas	748
S.2.5-2. Bessel functions for $\nu = \pm n \pm \frac{1}{2}$, where $n = 0, 1, 2, \dots$	748
S.2.5-3. Bessel functions for $\nu = \pm n$, where $n = 0, 1, 2, \dots$	749
S.2.5-4. Wronskians and similar formulas	749
S.2.5-5. Integral representations	749
S.2.5-6. Asymptotic expansions	750
S.2.5-7. Zeros and orthogonality properties of the Bessel functions	750
S.2.5-8. Hankel functions (Bessel functions of the third kind)	750
S.2.6. Modified Bessel Functions	751
S.2.6-1. Definitions. Basic formulas	751
S.2.6-2. Modified Bessel functions for $\nu = \pm n \pm \frac{1}{2}$, where $n = 0, 1, 2, \dots$	751
S.2.6-3. Modified Bessel functions for $\nu = n$, where $n = 0, 1, 2, \dots$	752
S.2.6-4. Wronskians and similar formulas	752
S.2.6-5. Integral representations	752
S.2.6-6. Asymptotic expansions as $x \rightarrow \infty$	752
S.2.7. Degenerate Hypergeometric Functions	753
S.2.7-1. Definitions. The Kummer's series	753
S.2.7-2. Some transformations and linear relations	753
S.2.7-3. Differentiation formulas and Wronskian	753
S.2.7-4. Degenerate hypergeometric functions for $n = 0, 1, 2, \dots$	754
S.2.7-5. Integral representations	754
S.2.7-6. Asymptotic expansion as $ x \rightarrow \infty$	754
S.2.7-7. Whittaker functions	755
S.2.8. Hypergeometric Functions	755
S.2.8-1. Definition. The hypergeometric series	755
S.2.8-2. Basic properties	755
S.2.8-3. Integral representations	755
S.2.9. Legendre Functions and Legendre Polynomials	756
S.2.9-1. Definitions. Basic formulas	756
S.2.9-2. Trigonometric expansions	756
S.2.9-3. Some relations	757
S.2.9-4. Integral representations	757
S.2.9-5. Legendre polynomials	757
S.2.9-6. Zeros of the Legendre polynomials and the generating function	758
S.2.9-7. Associated Legendre functions	758
S.2.10. Parabolic Cylinder Functions	758
S.2.10-1. Definitions. Basic formulas	758
S.2.10-2. Integral representations	759
S.2.10-3. Asymptotic expansion as $ z \rightarrow \infty$	759
S.2.11. Orthogonal Polynomials	759
S.2.11-1. Laguerre polynomials and generalized Laguerre polynomials	759
S.2.11-2. Chebyshev polynomials	760
S.2.11-3. Hermite polynomials	761

S.2.11-4. Gegenbauer polynomials	762
S.2.11-5. Jacobi polynomials	762
S.2.12. The Weierstrass Function	762
S.2.12-1. Definitions	762
S.2.12-2. Some properties	762
S.3. Tables of Indefinite Integrals	763
S.3.1. Integrals Containing Rational Functions	763
S.3.1-1. Integrals containing $a + bx$	763
S.3.1-2. Integrals containing $a + x$ and $b + x$	763
S.3.1-3. Integrals containing $a^2 + x^2$	764
S.3.1-4. Integrals containing $a^2 - x^2$	765
S.3.1-5. Integrals containing $a^3 + x^3$	765
S.3.1-6. Integrals containing $a^3 - x^3$	766
S.3.1-7. Integrals containing $a^4 \pm x^4$	766
S.3.2. Integrals Containing Irrational Functions	767
S.3.2-1. Integrals containing $x^{1/2}$	767
S.3.2-2. Integrals containing $(a + bx)^{p/2}$	767
S.3.2-3. Integrals containing $(x^2 + a^2)^{1/2}$	768
S.3.2-4. Integrals containing $(x^2 - a^2)^{1/2}$	768
S.3.2-5. Integrals containing $(a^2 - x^2)^{1/2}$	768
S.3.2-6. Reduction formulas	769
S.3.3. Integrals Containing Exponential Functions	769
S.3.4. Integrals Containing Hyperbolic Functions	769
S.3.4-1. Integrals containing $\cosh x$	769
S.3.4-2. Integrals containing $\sinh x$	770
S.3.4-3. Integrals containing $\tanh x$ or $\coth x$	771
S.3.5. Integrals Containing Logarithmic Functions	772
S.3.6. Integrals Containing Trigonometric Functions	773
S.3.6-1. Integrals containing $\cos x$	773
S.3.6-2. Integrals containing $\sin x$	774
S.3.6-3. Integrals containing $\sin x$ and $\cos x$	776
S.3.6-4. Reduction formulas	776
S.3.6-5. Integrals containing $\tan x$ and $\cot x$	776
S.3.7. Integrals Containing Inverse Trigonometric Functions	777
References	779
Index	783