



1. Linear Difference and Functional Equations with One Independent Variable

1.1. Linear Difference and Functional Equations Containing Unknown Function with Two Different Arguments

First-order linear difference equations.

1. $y(x + 1) - \alpha y(x) = 0.$

First-order constant-coefficient linear homogeneous difference equation.

2. $y(x + 1) - \alpha y(x) = f(x).$

First-order constant-coefficient linear nonhomogeneous difference equation.

3. $y(x + 1) - xy(x) = 0.$

4. $y(x + 1) - a(x - b)(x - c)y(x) = 0.$

5. $y(x + 1) - R(x)y(x) = 0, \quad R(x) = a \frac{(x - \lambda_1)(x - \lambda_2) \dots (x - \lambda_n)}{(x - \mu_1)(x - \mu_2) \dots (x - \mu_m)}.$

6. $y(x + 1) - f(x)y(x) = 0.$

7. $y(x + a) - by(x) = 0.$

8. $y(x + a) - by(x) = f(x).$

9. $y(x + a) - bxy(x) = 0, \quad a, b > 0.$

10. $y(x + a) - f(x)y(x) = 0.$

Linear functional equations containing $y(x)$ and $y(ax)$.

11. $y(ax) - by(x) = 0, \quad a, b > 0.$

12. $y(ax) - by(x) = f(x).$

Linear functional equations containing $y(x)$ and $y(a - x)$.

13. $y(x) - y(a - x) = 0.$

14. $y(x) + y(a - x) = 0.$

15. $y(x) + y(a - x) = b.$

16. $y(x) + y(a - x) = f(x).$

17. $y(x) - y(a - x) = f(x).$

18. $y(x) + g(x)y(a - x) = f(x).$

Linear functional equations containing $y(x)$ and $y(z)$, where $z = \varphi(x)$.

19. $y(x^a) - by(x) = 0, \quad a, b > 0.$
20. $y(x) - y(a/x) = 0.$
21. $y(x) + y(a/x) = 0.$
22. $y(x) + y(a/x) = b.$
23. $y(x) + y(a/x) = f(x).$
24. $y(x) - y(a/x) = f(x).$
25. $y(x) + g(x)y(a/x) = f(x).$
26. $y(x) - y\left(\frac{a-x}{1+bx}\right) = 0.$
27. $y(x) + y\left(\frac{a-x}{1+bx}\right) = 0.$
28. $y(x) + y\left(\frac{a-x}{1+bx}\right) = f(x).$
29. $y(x) - y\left(\frac{a-x}{1+bx}\right) = f(x).$
30. $y(x) - cy\left(\frac{a-x}{1+bx}\right) = f(x), \quad c \neq \pm 1.$
31. $y(x) + g(x)y\left(\frac{a-x}{1+bx}\right) = f(x).$
32. $y(x) + cy\left(\frac{ax-\beta}{x+b}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
33. $y(x) + cy\left(\frac{bx+\beta}{a-x}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
34. $y(x) + g(x)y\left(\frac{ax-\beta}{x+b}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
35. $y(x) + g(x)y\left(\frac{bx+\beta}{a-x}\right) = f(x), \quad \beta = a^2 + ab + b^2.$
36. $y(x) - y(\sqrt{a^2 - x^2}) = 0, \quad 0 \leq x \leq a.$
37. $y(x) + y(\sqrt{a^2 - x^2}) = 0, \quad 0 \leq x \leq a.$
38. $y(x) + y(\sqrt{a^2 - x^2}) = b, \quad 0 \leq x \leq a.$
39. $y(x) + y(\sqrt{a^2 - x^2}) = f(x), \quad 0 \leq x \leq a.$
40. $y(x) - y(\sqrt{a^2 - x^2}) = f(x), \quad 0 \leq x \leq a.$
41. $y(x) + g(x)y(\sqrt{a^2 - x^2}) = f(x), \quad 0 \leq x \leq a.$

Linear functional equations containing $y(\sin x)$ and $y(\cos x)$.

42. $y(\sin x) - y(\cos x) = 0$.
43. $y(\sin x) + y(\cos x) = 0$.
44. $y(\sin x) + y(\cos x) = a$.
45. $y(\sin x) + y(\cos x) = f(x)$.
46. $y(\sin x) - y(\cos x) = f(x)$.
47. $y(\sin x) + g(x)y(\cos x) = f(x)$.

Linear functional equations containing $y(x)$ and $y(\omega(x))$, where $\omega(\omega(x)) = x$.

48. $y(x) - y(\omega(x)) = 0$, where $\omega(\omega(x)) = x$.
49. $y(x) + y(\omega(x)) = 0$, where $\omega(\omega(x)) = x$.
50. $y(x) + y(\omega(x)) = b$, where $\omega(\omega(x)) = x$.
51. $y(x) + y(\omega(x)) = f(x)$, where $\omega(\omega(x)) = x$.
52. $y(x) - y(\omega(x)) = f(x)$, where $\omega(\omega(x)) = x$.
53. $y(x) + g(x)y(\omega(x)) = f(x)$, where $\omega(\omega(x)) = x$.

1.2. Other Linear Difference and Functional Equations**Second-order linear difference equations, $y_n = y(n)$.**

1. $y_{n+2} + ay_{n+1} + by_n = 0$.
Second-order constant-coefficient linear homogeneous difference equation.
2. $y_{n+2} + ay_{n+1} + by_n = f_n$.
Second-order constant-coefficient linear nonhomogeneous difference equation.
3. $y(x+2) + ay(x+1) + by(x) = 0$.
Second-order constant-coefficient linear homogeneous difference equation.
4. $y(x+2) + ay(x+1) + by(x) = f(x)$.
Second-order constant-coefficient linear nonhomogeneous difference equation.
5. $y(x+2) + a(x+1)y(x+1) + bx(x+1)y(x) = 0$.

Other functional equations.

6. $Ay(ax) + By(bx) + y(x) = 0$.
7. $Ay(x^a) + By(x^b) + y(x) = 0$.
8. $y(y(x)) - x = 0$.
9. $y(y(x)) + ay(x) + bx = 0$.

10. $y(y(y(x))) - x = 0$.
11. $Ay(x) + By\left(\frac{ax - \beta}{x + b}\right) + Cy\left(\frac{bx + \beta}{a - x}\right) = f(x), \quad \beta = a^2 + ab + b^2$.
12. $f_1(x)y(x) + f_2(x)y\left(\frac{ax - \beta}{x + b}\right) + f_3(x)y\left(\frac{bx + \beta}{a - x}\right) = g(x), \quad \beta = a^2 + ab + b^2$.
13. $y_{n+m} + a_{m-1}y_{n+m-1} + \dots + a_1y_{n+1} + a_0y_n = 0$.
*m*th-order constant-coefficient linear nonhomogeneous difference equation.
14. $y_{n+m} + a_{m-1}y_{n+m-1} + \dots + a_1y_{n+1} + a_0y_n = f_n$.
*m*th-order constant-coefficient linear nonhomogeneous difference equation.
15. $y(x + n) + a_{n-1}y(x + n - 1) + \dots + a_1y(x + 1) + a_0y(x) = 0$.
*n*th-order constant-coefficient linear homogeneous difference equation.
16. $y(x + n) + a_{n-1}y(x + n - 1) + \dots + a_1y(x + 1) + a_0y(x) = f(x)$.
*n*th-order constant-coefficient linear nonhomogeneous difference equation.
17. $y(x + b_n) + a_{n-1}y(x + b_{n-1}) + \dots + a_1y(x + b_1) + a_0y(x) = 0$.
18. $y(x^{a_n}) + b_{n-1}y(x^{a_{n-1}}) + \dots + b_1y(x^{a_1}) + b_0y(x) = 0$.
19. $y(a_n x) + b_{n-1}y(a_{n-1}x) + \dots + b_1y(a_1x) + b_0y(x) = 0$.
20. $y^{[n]}(x) + a_{n-1}y^{[n-1]}(x) + \dots + a_1y(x) + a_0x = 0, \quad y^{[n]}(x) = y(y^{[n-1]}(x))$.

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.