4. Nonlinear Functional Equations with Several Independent Variables

1. \( f(x + y) = f(x)f(y) \). \textit{Exponential Cauchy equation.}

2. \( f(xy) = f(x)f(y) \). \textit{Power-law Cauchy equation.}

3. \( f(y + x) + f(y - x) = 2f(x)f(y) \). \textit{D’Alembert equation.}

4. \( f(x + y) = a^{xy}f(x)f(y) \).

5. \( f(x + y) = f(x) + f(y) - af(x)f(y) \). \textit{Equation of the theory probability.}

6. \( f(x)g(y) = h(x + y) \).

7. \( f(x)g(y) + h(y) = f(x + y) \).

8. \( f(x + y)f(x - y) = f^2(x) \). \textit{Lobachevsky equation.}

9. \( \frac{f^2(x) + f^2(y)}{2} \left(\frac{x^2 + y^2}{2}\right)^{1/2} \).

10. \( f(x, y)f(y, z) = f(x, z) \).

11. \( M(f(x), f(y)) = f(M(x, y)) \).

12. \( f_1(x)g_1(y) + f_2(x)g_2(y) + f_3(x)g_3(y) = 0 \). \textit{Bilinear functional equation.}

13. \( f_1(x)g_1(y) + f_2(x)g_2(y) + f_3(x)g_3(y) + f_4(x)g_4(y) = 0 \). \textit{Bilinear functional equation.}

14. \( f(x) + g(y) = Q(z) \), where \( z = \varphi(x) + \psi(y) \).

15. \( f(t) + g(x) + h(x)Q(z) + R(z) = 0 \), where \( z = \varphi(x) + \psi(t) \).

16. \( f(t) + g(x)Q(z) + h(x)R(z) = 0 \), where \( z = \varphi(x) + \psi(t) \).

The EqWorld website presents extensive information on solutions to various classes of ordinary differential equations, partial differential equations, integral equations, functional equations, and other mathematical equations.

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