



1.  $y(x + 1) - ay(x) = 0$ .

*First-order constant-coefficient linear homogeneous difference equation.*

Solution:

$$y(x) = \Theta(x)a^x,$$

where  $\Theta(x) = \Theta(x + 1)$  is an arbitrary periodic function with unit period. If  $\Theta(x) \equiv \text{const}$ , there is a particular solution  $y(x) = Ca^x$ , where  $C$  is an arbitrary constant.

*Remark.* Here and henceforth in similar cases,  $\Theta(x)$  can be an arbitrary periodic function with period  $T = \frac{1}{n}$ , where  $n$  is any positive integer. To the minimum value  $n = 1$  there corresponds a function with maximum period  $T_{\max} = 1$ .

### References

- Kuczma, M.**, *Functional Equations in a Single Variable*, Polish Scientific Publishers, 1968.  
**Miroyubov, A. A., and Soldatov, M. A.**, *Linear Homogeneous Difference Equations* [in Russian], Nauka, Moscow, 1981.  
**Polyanin, A. D. and Manzhirov, A. V.**, *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.