3. $y(x+1) - xy(x) = 0.$

Solution:

$$y(x) = \Theta(x)\Gamma(x), \qquad \Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt,$$

where $\Gamma(x)$ is the gamma function, $\Theta(x) = \Theta(x+1)$ is an arbitrary periodic function with unit period.

The simplest particular solution corresponds to $\Theta(x) \equiv 1$.

References

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