



3. $y(x+1) - xy(x) = 0$.

Solution:

$$y(x) = \Theta(x)\Gamma(x), \quad \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt,$$

where $\Gamma(x)$ is the gamma function, $\Theta(x) = \Theta(x+1)$ is an arbitrary periodic function with unit period. The simplest particular solution corresponds to $\Theta(x) \equiv 1$.

References

Miroyubov, A. A., and Soldatov, M. A., *Linear Homogeneous Difference Equations* [in Russian], Nauka, Moscow, 1981 (page 46).

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.