8. $y(x + a) - by(x) = f(x)$.

1°. Solution:

$$y(x) = \Theta(x)b^{x/a} + \bar{y}(x),$$

where $\Theta(x) = \Theta(x + a)$ is an arbitrary periodic function with period $a$, and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. For $f(x) = \sum_{k=0}^{n} a_k x^n$ and $b \neq 1$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=0}^{n} B_k x^n$, where the constants $B_k$ are found by the method of undetermined coefficients.

3°. For $f(x) = \sum_{k=1}^{n} A_k \exp(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \exp(\lambda_k x)$, where the constants $B_k$ are found by the method of undetermined coefficients.

4°. For $f(x) = \sum_{k=1}^{n} A_k \cos(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

5°. For $f(x) = \sum_{k=1}^{n} A_k \sin(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

Reference