8. $y(x+a) - by(x) = f(x)$.

1°. Solution:

$$y(x) = \Theta(x) b^{x/a} + \bar{y}(x),$$

where $\Theta(x) = \Theta(x+a)$ is an arbitrary periodic function with period a, and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. For $f(x) = \sum_{k=0}^{n} A_k x^n$ and $b \neq 1$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=0}^{n} B_k x^n$, where the constants B_k are found by the method of undetermined coefficients.

3°. For $f(x) = \sum_{k=1}^{n} A_k \exp(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \exp(\lambda_k x)$, where the constants B_k are found by the method of undetermined coefficients.

4°. For $f(x) = \sum_{k=1}^{n} A_k \cos(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants B_k and D_k are found by the method of undetermined coefficients.

5°. For $f(x) = \sum_{k=1}^{n} A_k \sin(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants B_k and D_k are found by the method of undetermined coefficients.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

