12. $y(ax) - by(x) = f(x)$.

1°. Solution:

$$y(x) = Y(x) + \bar{y}(x),$$

where $Y(x)$ is the general solution of the homogeneous equation $Y(ax) - bY(x) = 0$ (see the previous equation 11), and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. For $f(x) = \sum_{k=0}^{n} A_k x^n$, the nonhomogeneous equation has a particular solution

$$\bar{y}(x) = \sum_{k=0}^{n} \frac{A_k}{a^k - b} x^k, \qquad a^k - b \neq 0.$$

3°. For $f(x) = \ln x \sum_{k=0}^{n} A_k x^k$, the nonhomogeneous equation has a particular solution

$$\bar{y}(x) = \sum_{k=1}^{n} x^k \left(B_k \ln x + C_k\right), \qquad B_k = \frac{A_k}{a^k - b}, \quad C_k = -\frac{A_k a^k \ln a}{(a^k - b)^2}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

