



**12.  $y(ax) - by(x) = f(x)$ .**

1°. Solution:

$$y(x) = Y(x) + \bar{y}(x),$$

where  $Y(x)$  is the general solution of the homogeneous equation  $Y(ax) - bY(x) = 0$  (see the previous equation 11), and  $\bar{y}(x)$  is any particular solution of the nonhomogeneous equation.

2°. For  $f(x) = \sum_{k=0}^n A_k x^k$ , the nonhomogeneous equation has a particular solution

$$\bar{y}(x) = \sum_{k=0}^n \frac{A_k}{a^k - b} x^k, \quad a^k - b \neq 0.$$

3°. For  $f(x) = \ln x \sum_{k=0}^n A_k x^k$ , the nonhomogeneous equation has a particular solution

$$\bar{y}(x) = \sum_{k=1}^n x^k (B_k \ln x + C_k), \quad B_k = \frac{A_k}{a^k - b}, \quad C_k = -\frac{A_k a^k \ln a}{(a^k - b)^2}.$$

### Reference

**Polyanin, A. D. and Manzhirov, A. V.**, *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.