13. $y(x) - y(a - x) = 0.$

Solution:

$$y(x) = \Phi(x, a - x),$$

where $\Phi(x, z) = \Phi(z, x)$ is any symmetric function with two arguments.

As $\Phi(x, z)$, one may take $\Phi(x, z) = \varphi(x, z) + \varphi(z, x)$, where $\varphi(x, z)$ is any function of two arguments. A special case of this formula is $\Phi(x, z) = \varphi(x) + \varphi(z)$, where $\varphi(x)$ is an arbitrary function of a single argument.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

