15. $y(x) + y(a - x) = b.$

Solution:

$$y(x) = \tfrac{1}{2}b + \Phi(x, a - x),$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function with two arguments.

Particular solutions:

$$y(x) = b\sin^2\left(\frac{\pi x}{2a}\right),$$

$$y(x) = b\cos^2\left(\frac{\pi x}{2a}\right).$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

