31. $y(x) + g(x)y\left(\dfrac{a-x}{1+bx}\right) = f(x).$

Solution:

$$y(x) = \frac{f(x) - g(x)f(z)}{1 - g(x)g(z)}, \qquad z = \frac{a-x}{1+bx}.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

