



39. $y(x) + y(\sqrt{a^2 - x^2}) = f(x), \quad 0 \leq x \leq a.$

The function $f(x)$ is assumed to satisfy the condition $f(x) = f(\sqrt{a^2 - x^2})$.

Solution:

$$y(x) = \frac{1}{2}f(x) + \Phi(x, \sqrt{a^2 - x^2}),$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

References

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.