



51. $y(x) + y(\omega(x)) = f(x)$, **where** $\omega(\omega(x)) = x$.

The function $f(x)$ is assumed to satisfy the condition $f(x) = f(\omega(x))$.

Solution:

$$y(x) = \frac{1}{2}f(x) + \Phi(x, \omega(x)),$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.