4. $y(x + 2) + ay(x + 1) + by(x) = f(x)$.

**Second-order constant-coefficient linear nonhomogeneous difference equation.**

1°. Solution:

$$y(x) = Y(x) + ar{y}(x),$$

where $Y(x)$ is the general solution of the corresponding homogeneous equation $Y(x + 2)aY(x + 1) + bY(x) = 0$ (see the preceding equation), and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. If $f(x) = \sum_{k=0}^{n} A_k x^n$ and $a + b + 1 \neq 1$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=0}^{n} B_k x^n$, where the constants $B_k$ are found by the method of undetermined coefficients.

3°. If $f(x) = \sum_{k=0}^{n} A_k \exp(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \exp(\lambda_k x)$, where the constants $B_k$ are found by the method of undetermined coefficients.

4°. If $f(x) = \sum_{k=1}^{n} A_k \cos(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

5°. If $f(x) = \sum_{k=1}^{n} A_k \sin(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x) = \sum_{k=1}^{n} B_k \cos(\lambda_k x) + \sum_{k=1}^{n} D_k \sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

**Reference**

