## 4. $y(x+2)+ay(x+1)+by(x)=f(x)$.

***Second-order constant-coefficient linear nonhomogeneous difference equation.***

1°. Solution:

$$y(x)=Y(x)+\bar{y}(x),$$

where $Y(x)$ is the general solution of the corresponding homogeneous equation $Y(x+2)aY(x+1)+bY(x)=0$ (see the preceding equation), and $\bar{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. If $f(x)=\sum\limits_{k=0}^{n}A_k x^n$ and $a+b+1\neq 1$, the nonhomogeneous equation has a particular solution $\bar{y}(x)=\sum\limits_{k=0}^{n}B_k x^n$, where the constants $B_k$ are found by the method of undetermined coefficients.

3°. If $f(x)=\sum\limits_{k=1}^{n}A_k\exp(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x)=\sum\limits_{k=1}^{n}B_k\exp(\lambda_k x)$, where the constants $B_k$ are found by the method of undetermined coefficients.

4°. If $f(x)=\sum\limits_{k=1}^{n}A_k\cos(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x)=\sum\limits_{k=1}^{n}B_k\cos(\lambda_k x)+\sum\limits_{k=1}^{n}D_k\sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

5°. If $f(x)=\sum\limits_{k=1}^{n}A_k\sin(\lambda_k x)$, the nonhomogeneous equation has a particular solution $\bar{y}(x)=\sum\limits_{k=1}^{n}B_k\cos(\lambda_k x)+\sum\limits_{k=1}^{n}D_k\sin(\lambda_k x)$, where the constants $B_k$ and $D_k$ are found by the method of undetermined coefficients.

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