9. $\boldsymbol{y\big(y(x)\big) + ay(x) + bx = 0.}$

General solution in parametric form:

$$x = C_1(t)\lambda_1^t + C_2(t)\lambda_2^t,$$
$$y = C_1(t)\lambda_1^{t+1} + C_2(t)\lambda_2^{t+1},$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 + a\lambda + b = 0$$

and $C_1 = C_1(t)$ and $C_2 = C_2(t)$ are arbitrary periodic functions with unit period, $C_n(t) = C_n(t+1)$.

For $C_1 = \text{const}$ and $C_2 = \text{const}$, there is a particular solution that can be written out in implicit form as

$$\frac{\lambda_2 x - y(x)}{\lambda_2 - \lambda_1} = C_1\left[\frac{\lambda_1 x - y(x)}{C_2(\lambda_1 - \lambda_2)}\right]^k, \qquad k = \frac{\ln\lambda_1}{\ln\lambda_2},$$

where C is an arbitrary constant.

References

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