11. $Ay(x) + By\left(\dfrac{ax-\beta}{x+b}\right) + Cy\left(\dfrac{bx+\beta}{a-x}\right) = f(x), \qquad \beta = a^2 + ab + b^2.$

In the equation, let us substitute first $\dfrac{ax-\beta}{x+b}$ for x and then $\dfrac{bx+\beta}{a-x}$ for x to obtain (the original equation comes first)

$$\begin{aligned} Ay(x) + By(u) + Cy(w) &= f(x),\\ Ay(u) + By(w) + Cy(x) &= f(u),\\ Ay(w) + By(x) + Cy(u) &= f(w), \end{aligned} \tag{1}$$

where $u = \dfrac{ax-\beta}{x+b}$ and $w = \dfrac{bx+\beta}{a-x}$. Eliminating $y(u)$ and $y(w)$ from the system of linear algebraic equations (1), we arrive at a solution of the original functional equation.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

