



11.  $Ay(x) + By\left(\frac{ax - \beta}{x + b}\right) + Cy\left(\frac{bx + \beta}{a - x}\right) = f(x), \quad \beta = a^2 + ab + b^2.$

In the equation, let us substitute first  $\frac{ax - \beta}{x + b}$  for  $x$  and then  $\frac{bx + \beta}{a - x}$  for  $x$  to obtain (the original equation comes first)

$$\begin{aligned} Ay(x) + By(u) + Cy(w) &= f(x), \\ Ay(u) + By(w) + Cy(x) &= f(u), \\ Ay(w) + By(x) + Cy(u) &= f(w), \end{aligned} \tag{1}$$

where  $u = \frac{ax - \beta}{x + b}$  and  $w = \frac{bx + \beta}{a - x}$ . Eliminating  $y(u)$  and  $y(w)$  from the system of linear algebraic equations (1), we arrive at a solution of the original functional equation.

### Reference

**Polyanin, A. D. and Manzhirov, A. V.**, *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.