13. \( y_{n+m} + a_{m-1}y_{n+m-1} + \ldots + a_1y_{n+1} + a_0y_n = 0. \)

This is an \( m \)th-order linear homogeneous difference equation defined on a discrete set of points \( x = 0, 1, 2, \ldots \) The notation \( y_n = y(n) \) is used.

Suppose \( \lambda_1, \lambda_2, \ldots, \lambda_m \) are roots of the characteristic equation

\[
P(\lambda) \equiv \lambda^m + a_{m-1}\lambda^{m-1} + \ldots + a_1\lambda + a_0 = 0. \tag{1}
\]

If the roots of equation (1) are all different, the general solution of the difference equation is expressed as

\[
y_n = \sum_{i=0}^{m-1} y_i \sum_{j=0}^{m-i-1} a_{i+j+1} \sum_{k=1}^{m} \frac{\lambda_k^{n+1}}{P'(\lambda_k)}, \tag{2}
\]

where the prime stands for differentiation.

The initial values \( y_0, y_1, \ldots, y_m \) that occur in formula (2) can be set arbitrarily.

In the case of complex conjugate roots in solution (2), the real and imaginary parts should be separated.

References


