**13.** $y_{n+m} + a_{m-1}y_{n+m-1} + \dots + a_1 y_{n+1} + a_0 y_n = 0.$

This is an ***$m$th-order linear homogeneous difference equation*** defined on a discrete set of points $x = 0, 1, 2, \dots$ The notation $y_n = y(n)$ is used.

Suppose $\lambda_1, \lambda_2, \dots, \lambda_m$ are roots of the characteristic equation

$$P(\lambda) \equiv \lambda^m + a_{m-1}\lambda^{m-1} + \dots + a_1\lambda + a_0 = 0. \tag{1}$$

If the roots of equation (1) are all different, the general solution of the difference equation is expressed as

$$y_n = \sum_{i=0}^{m-1} y_i \sum_{j=0}^{m-i-1} a_{i+j+1} \sum_{k=1}^{m} \frac{\lambda_k^{n+1}}{P'(\lambda_k)}, \tag{2}$$

where the prime stands for differentiation.

The initial values $y_0, y_1, \dots, y_m$ that occur in formula (2) can be set arbitrarily.

In the case of complex conjugate roots in solution (2), the real and imaginary parts should be separated.

## References

**Kuczma, M.,** *Functional Equations in a Single Variable*, Polish Scientific Publishers, 1968.

**Doetsch, G.,** *Guide to the Applications of the Laplace and $Z$-Transforms* [in Russian], Nauka, Moscow, 1971 (page 213); English edition: Van Nostrand Reinhold Co., London, 1971.

**Mirolyubov, A. A., and Soldatov, M. A.,** *Linear Homogeneous Difference Equations* [in Russian], Nauka, Moscow, 1981.

**Polyanin, A. D. and Manzhirov, A. V.,** *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.



 