14. $y_{n+m} + a_{m-1}y_{n+m-1} + \dots + a_1y_{n+1} + a_0y_n = f_n.$

This is an ***mth-order linear nonhomogeneous difference equation*** defined on a discrete set of points $x = 0, 1, 2, \ldots$ The notation $y_n = y(n)$, $f_n = f(n)$ is used.

The general solution of the difference equation has the form $y(x) = Y(x) + \bar{y}(x)$, where $Y(x)$ is the general solution of the homogeneous equation (with $f_n \equiv 0$) and $\bar{y}(x)$ is an particular solution of the nonhomogeneous equation.

Suppose $\lambda_1, \lambda_2, \ldots, \lambda_m$ are roots of the characteristic equation

$$P(\lambda) \equiv \lambda^m + a_{m-1}\lambda^{m-1} + \dots + a_1\lambda + a_0 = 0. \qquad (1)$$

If the roots of equation (1) are all different, the general solution of the difference equation is expressed as

$$y_n = \sum_{i=0}^{m-1} y_i \sum_{j=0}^{m-i-1} a_{i+j+1} \sum_{k=1}^{m} \frac{\lambda_k^{n+1}}{P'(\lambda_k)} + \sum_{\nu=m}^{n} f_{n-\nu} \sum_{k=1}^{m} \frac{\lambda_k^{\nu-1}}{P'(\lambda_k)}, \qquad (2)$$

where the prime stands for differentiation.

The initial values $y_0, y_1, \ldots, y_m$ that occur in formula (2) can be set arbitrarily.

In the case of complex conjugate roots in solution (2), the real and imaginary parts should be separated.

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