16. $y(x+n)+a_{n-1}y(x+n-1)+\ldots+a_1y(x+1)+a_0y(x)=f(x).$

nth-order constant-coefficient linear nonhomogeneous difference equation.

1°. Solution:

$$y(x)=Y(x)+\overline{y}(x),$$

where $Y(x)$ is the general solution of the homogeneous equation,

$$Y(x+n)+a_{n-1}Y(x+n-1)+\ldots+a_1Y(x+1)+a_0Y(x)=0$$

(see the preceding equation), and $\overline{y}(x)$ is any particular solution of the nonhomogeneous equation.

2°. For $f(x)=\sum\limits_{k=0}^{n}A_kx^n$, the nonhomogeneous equation has a particular solution $\overline{y}(x)=\sum\limits_{k=0}^{n}B_kx^n$, where the constants B_k are found by the method of undetermined coefficients.

3°. For $f(x)=\sum\limits_{k=1}^{n}A_k\exp(\lambda_kx)$, the nonhomogeneous equation has a particular solution $\overline{y}(x)=\sum\limits_{k=1}^{n}B_k\exp(\lambda_kx)$, where the constants B_k are found by the method of undetermined coefficients.

4°. For $f(x)=\sum\limits_{k=1}^{n}A_k\cos(\lambda_kx)$, the nonhomogeneous equation has a particular solution $\overline{y}(x)=\sum\limits_{k=1}^{n}B_k\cos(\lambda_kx)+\sum\limits_{k=1}^{n}D_k\sin(\lambda_kx)$, where the constants B_k and D_k are found by the method of undetermined coefficients.

5°. For $f(x)=\sum\limits_{k=1}^{n}A_k\sin(\lambda_kx)$, the nonhomogeneous equation has a particular solution $\overline{y}(x)=\sum\limits_{k=1}^{n}B_k\cos(\lambda_kx)+\sum\limits_{k=1}^{n}D_k\sin(\lambda_kx)$, where the constants B_k and D_k are found by the method of undetermined coefficients.

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