



20. $y^{[n]}(x) + a_{n-1}y^{[n-1]}(x) + \dots + a_1y(x) + a_0x = 0.$

Notation: $y^{[2]}(x) = y(y(x)), \dots, y^{[n]}(x) = y(y^{[n-1]}(x)).$

Solutions may be sought in the form

$$x = w(t), \quad y = w(t + 1).$$

Then the original equation is reduced to the following n th-order linear finite-difference equation (see equation 15 in the current subsection):

$$w(t + n) + a_{n-1}w(t + n - 1) + \dots + a_1w(t + 1) + a_0w(t) = 0.$$

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.