



5. $y(x)y(a-x) = f^2(x)$.

The right-hand side function is assumed to satisfy the condition $f(x) = \pm f(a-x)$. To be specific, we assume $f(x) = f(a-x)$.

Solution:

$$y(x) = \pm f(x) \exp[\Phi(x, a-x)],$$

where $\Phi(x, z) = -\Phi(z, x)$ is any antisymmetric function of two arguments.

For $\Phi(x, z) = C(x-z)$, there are particular solutions of the form

$$y(x) = \pm e^{C(2x-a)} f(x),$$

where C is an arbitrary constant.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.