18. $y(x)y\left(\sqrt{a^2-x^2}\right) = f^2(x), \qquad 0 \le x \le a.$

The right-hand side function is assumed to satisfy the condition $f(x) = \pm f\left(\sqrt{a^2-x^2}\right)$. To be specific, we take $f(x) = f\left(\sqrt{a^2-x^2}\right)$.

Solution:

$$y(x) = \pm f(x)\exp\left[\Phi\left(x,\ \sqrt{a^2-x^2}\right)\right],$$

where $\Phi(x,z) = -\Phi(z,x)$ is any antisymmetric function of two arguments.

Reference

Polyanin, A. D. and Manzhirov, A. V., *Handbook of Integral Equations: Exact Solutions (Supplement. Some Functional Equations)* [in Russian], Faktorial, Moscow, 1998.

