9. $f(x) + (1-x)f\left(\dfrac{y}{1-x}\right) = f(y) + (1-y)f\left(\dfrac{x}{1-y}\right).$

Basic equation of information theory. Here, x, y, $x+y$ can assume values from zero to one.

Solution:

$$f(x) = C[x\ln x + (1-x)\ln(1-x)],$$

where C is an arbitrary constant.

Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.

