



10. $f(1-x) + (1-x)^\alpha f\left(\frac{y}{1-x}\right) = f(y) + (1-y)^\alpha f\left(\frac{x}{1-y}\right).$

Here, $x, y, x+y$ can assume values from zero to one, and $\alpha \neq 0, 1, 2$.

Solution:

$$f(x) = C[x^\alpha + (1-x)^\alpha - 1],$$

where C is an arbitrary constant.

Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.