



4. $f(x + y) = a^{xy} f(x)f(y)$.

Solution:

$$f(x) = e^{Cx} a^{x^2/2},$$

where C is an arbitrary constant. In addition, the function $f(x) \equiv 0$ is also a solution.

Reference

Aczél, J. and Dhombres, J., *Functional Equations in Several Variables*, Cambridge Univ. Press, Cambridge, 1989.